香港城市大學
City University of Hong Kong

396EM Airline Operations and Scheduling/ 6075MAA Airline Scheduling and Operations

Lecture 3a
Fleet Assignment Part II

Developed & Revised :
Dr Yang Dai & Dr Ammar Bazi (Coventry)
Kimman Lui & Rossella Lau (Scope)
Presented: Kinki Leung

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Learning outcomes

Understand	the implementation of fleet assignment
Know	several key terms such as spill rate
Set up	the optimization formula including objective functions and constraints

Fight Schedule for Ultimate Air (Reference)
Total 42 flights

Table 3.3 Flight schedule for Ultimate Air

Flight no.	Origin	Departure time	Destination	Arrival time	Flight hours
101	LAX	05:00	JFK	13:30	5.5
104	SFO	05:05	JFK	13:35	5.5
116	BOS	06:15	JFK	07:45	1.5
140	JFK	06:20	IAD	07:20	1
125	JFK	07:25	SFO	09:55	5.5
107	ORD	07:30	JFK	10:30	2
122	JFK	07:35	LAX	10:05	5.5
137	JFK	07:40	BOS	09:10	1.5
110	ATL	08:10	JFK	10:40	2.5
119	IAD	08:15	JFK	09:15	1
113	MIA	09:10	JFK	12:10	3
131	JFK	09:30	ATL	12:00	2.5

Source: Airline Operation and Scheduling 2nd edition

Fight Schedule for Ultimate Air (Con't)

102	LAX	09:45	JFK	18:15	5.5
105	SFO	09:50	JFK	18:20	5.5
117	BOS	10:00	JFK	11:30	1.5
128	JFK	10:05	ORD	11:05	2
134	JFK	10:35	MIA	13:35	3
141	JFK	12:00	IAD	13:00	1
108	ORD	12:20	JFK	15:20	2
138	JFK	12:30	BOS	14:00	1.5
111	ATL	13:10	JFK	15:40	2.5
120	IAD	14:25	JFK	15:25	1
114	MIA	14:30	JFK	17:30	3
132	JFK	14:35	ATL	17:35	2.5
118	BOS	15:00	JFK	16:30	1.5
129	JFK	15:05	ORD	16:05	2
135	JFK	15:10	MIA	18:10	3

 Source: Airline Operation and Scheduling 2nd edition

Fight Schedule for Ultimate Air (Con't)

Flight no.	Origin	Departure time	Destination	Arrival time	Flight hours
142	JFK	15:15	IAD	16:15	1
103	LAX	15:20	JFK	23:50	5.5
106	SFO	15:25	JFK	23:55	5.5
126	JFK	15:30	SFO	18:00	5.5
123	JFK	16:00	LAX	18:30	5.5
109	ORD	17:10	JFK	20:10	2
112	ATL	18:00	JFK	20:30	2.5
133	JFK	18:05	ATL	20:35	2.5
136	JFK	18:10	MIA	21:10	3
115	MIA	18:15	JFK	21:15	3
121	IAD	18:30	JFK	19:30	1
124	JFK	19:00	LAX	21:30	5.5
127	JFK	20:00	SFO	22:30	5.5
130	JFK	21:00	ORD	22:00	2
139	JFK	21:30	BOS	23:00	1.5

Source: Airline Operation and Scheduling 2nd edition

Fleet Assignment

- Match each aircraft type in the fleet with a particular route in the schedule
- It concerns only fleet type, but not a particular aircraft
- Objective: assign as many flight segments as possible in a schedule to one or more fleet types, while optimising the objective function and meeting various operational constraints

Basic Terminology

- Operating Costs
  - Operating Costs of a flight = CASM x distance x number of seats on the aircraft
- Passenger-Spill Costs
  - Passenger-Spill is the degree of average demand, which exceeds the capacity offered. It means the revenue of lost passengers due to insufficient aircraft capacity
  - Also known as “Overflow Cost” or “Denied Boarding Cost”
- Recapture Rate
  - The recapture rate represents the percentage of passengers that were spilled but could be accommodated or recaptured on other flights by the same airline.

Fleet Assignment Tool-Time Space diagram

- Fleet assignment is keeping track of the fleet at different stations (airports) at any given point in time.

- A time-space network is developed to solve the fleet assignment problem

- In the time-space diagram, columns represent the different airports, and the times of the day are shown as rows.

Time-Space Diagram (Networks)

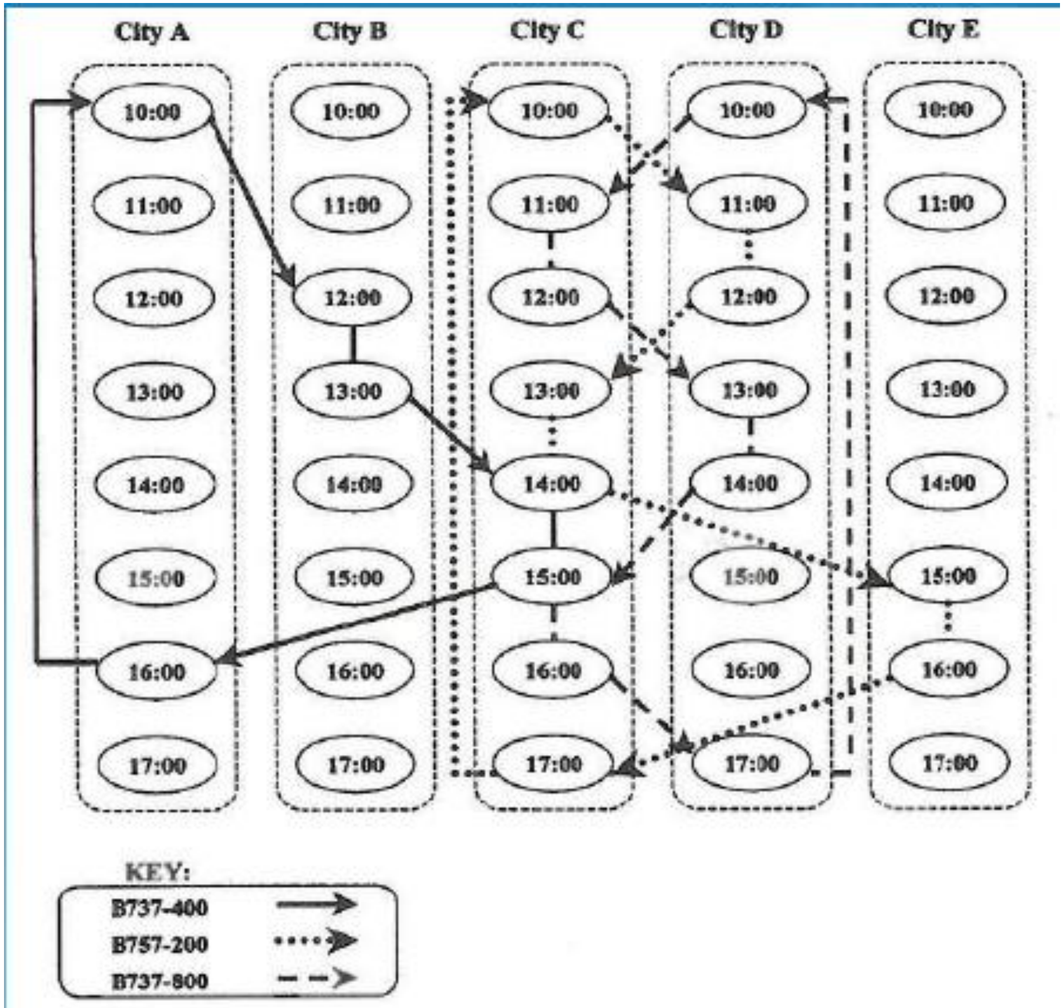
- Decisions that are needed to be made at different times require adding variables that keeps track of time.

- Time is a continuous variable!

- Adding a continuous variable to this problem makes the problem even more complicated to solve.

Sample of time-space diagram

- It presents the airports as columns, and times of the day as rows
- The arcs (arrows) are the flights, and nodes are the arrival and departure of a flight segment at a specific airport, at a specific time of the day.
- A wrap-around arc is a ground arc which connects the last node to the first node in a given city.
- These arcs represent the aircraft that stay overnight in an airport, and connect the last arrival to the next day's departure flight



Fleet Assignment problem

- Be viewed as the Multi-commodity network problem
- Each node represents supply/demand, which can be satisfied through a diverse fleet
- Objective: minimise the total cost or maximize the net profit by assigning the most appropriate fleet type to each flight leg.
- Constraints: each flight is assigned to a particular fleet type, and that the ***number of aircraft for each fleet does not exceed the number of available aircraft***
- Other side-constraints can include: curfew, range, noise, forced turns, maintenance, and user-specific restrictions

The objective function

- Minimize the total cost of the network

- There are 2 costs included:
  - operating costs
  - spill costs

Demand Information

Table 3.4 Destination in miles, demand means and standard deviations for Ultimate Air network

Flight no.	Origin	Destination	Distance (miles)	Demand	Standard deviation
101	LAX	JFK	2475	175	35
102	LAX	JFK	2475	182	36
103	LAX	JFK	2475	145	29
104	SFO	JFK	2586	178	35
105	SFO	JFK	2586	195	39
106	SFO	JFK	2586	162	32
107	ORD	JFK	740	165	33

Fleet Information – Operating Cost Example

- Assume that Ultimate Air has nine B737–800 and six B757–200 aircrafts.
- Seat capacities are 162 and 200 respectively.
- Cost per available seat mile (CASM) are $0.042 & $0.044 respectively
- Revenue per available seat mile (RASM) is $0.15.
- Operating cost for route between JFK and LAX in different fleets are?

Fleet Information – Operating Cost Example (Con't)

	Seats	CASM ($)	RASM ($)	Distance
B737–800	162	0.042	0.15	2,475 miles
B757–200	200	0.044	0.15	2,475 miles

- Operating cost (B737-800)

  = $0.042 x 2,475 x 162

  = $16,839.9

- Operating cost (B757-200)

  = $0.044 x 2,475 x 200

  = $21,780

Demand information (Con't)

Table 3.4 ***Continued***

Flight no.	Origin	Destination	Distance (miles)	Demand	Standard deviation
108	ORD	JFK	740	182	36
109	ORD	JFK	740	170	34
110	ATL	JFK	760	191	38
111	ATL	JFK	760	171	34
112	ATL	JFK	760	165	33
113	MIA	JFK	1090	198	39
114	MIA	JFK	1090	182	36
115	MIA	JFK	1090	168	33
116	BOS	JFK	187	115	23
117	BOS	JFK	187	146	29
118	BOS	JFK	187	120	24
119	IAD	JFK	228	135	27
120	IAD	JFK	228	109	21

Demand Information (Con't)

Table 3.4 ***Continued***

Flight no.	Origin	Destination	Distance (miles)	Demand	Standard deviation
121	IAD	JFK	228	98	19
122	JFK	LAX	2475	150	30
123	JFK	LAX	2475	145	29
124	JFK	LAX	2475	125	25
125	JFK	SFO	2586	148	29
126	JFK	SFO	2586	138	27
127	JFK	SFO	2586	121	24
128	JFK	ORD	740	132	26
129	JFK	ORD	740	129	25
130	JFK	ORD	740	117	23
131	JFK	ATL	760	168	33
132	JFK	ATL	760	160	32
133	JFK	ATL	760	191	38
134	JFK	MIA	1090	165	33
135	JFK	MIA	1090	184	36
136	JFK	MIA	1090	192	38

Demand Information (Con't)

Table 3.4 ***Concluded***

Flight no.	Origin	Destination	Distance (miles)	Demand	Standard deviation
137	JFK	BOS	187	147	29
138	JFK	BOS	187	135	27
139	JFK	BOS	187	146	29
140	JFK	IAD	228	105	21
141	JFK	IAD	228	115	23
142	JFK	IAD	228	118	23

Passenger spill costs

- Passenger demand is vital for effective fleet assignment since airline operation managers need to match the most suitable fleet type with the PAX demand on each flight segment

- Assigning large capacity aircraft to flights with low demand leads to low utilisation and consequently generates low load-factor for the airline

- However, assigning small aircraft to flight legs with high demand leads to passenger spills.

- Spill is the degree of average demand, which exceeds the capacity offered. The spill cost is the revenue of lost pax due to insufficient aircraft capacity

Example - Case study - Flight 122

- The historical data for flight 122 shows that the demand is normally distributed with a mean of 150 PAX and a SD of 30 PAX. (Table 3.4)

- The shaded areas show the probability of pax spills for the 2 fleet types: B737 & B757

- The spill is basically the truncation of the demand distribution beyond the aircraft capacity

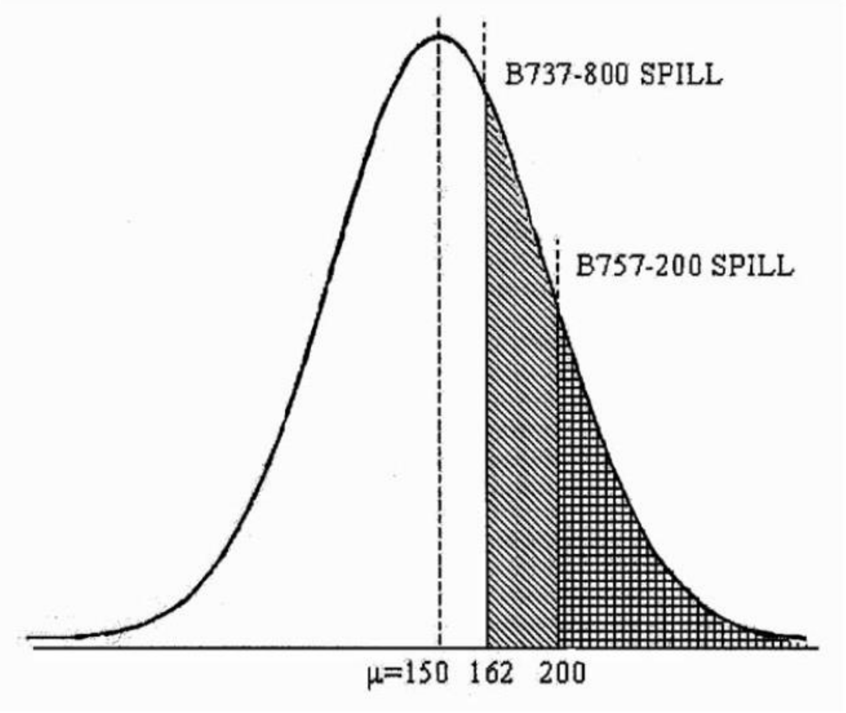


Figure 1: Demand distribution and passenger spills

Calculate the spill costs

$$\text{Expected spill cost for a fleet} = \text{expected number of passenger spill} \times \text{RASM} \times \text{distance}$$

The expected number of passenger spill is calculated as follows:

$$\text{Expected number of passenger spill} = \int_{c}^{\infty} (x-c) f(x)\,dx$$

- Where,
  - C: the fleet capacity
  - f(x) is the probability distribution function of the demand

Mathematical Software

- The integral can be obtained by using software (e.g. Excel or R).
- Example: Expected number of spilled passengers (B737-800 with 162 seats)
  - Cell A1: NORMINV (RAND(), 150, 30)
  - Cell B1: IF (A1>162, A1-162, 0)
- Cell A1 will randomly generates a demand from normal distribution with a mean of 150 and standard deviation of 30.
- Cell B1 checks if the demand in cell A1 exceeds 162 seats and assigned to their difference (i.e. passenger spill), otherwise it returns zero.
- Copy and paste the above two cells many times (e.g. 20,000 replications) and average column B (AVERAGE(B:B))
- Then we can get the expected numbers of passenger spill for the two fleet types

Using Excel to obtain the expected numbers of pax spills

- Expected passenger spill for B737–800 with 162 seat capacity = 6.91
- Expected passenger spill for B757–200 with 200 seat capacity = 0.60

➨ Using the following fleet spill cost equation:

$$\text{Expected spill cost for a fleet} = \text{expected number of passenger spill} \times \text{RASM} \times \text{distance}$$

➨ Then we can get the Expected Spill Costs (ESC) for both fleets: B737-800 and B757-200

ESC (B737-800) = 6.91*0.15*2475 = $2,565.34
ESC (B757-200) = 0.60*0.15*2475 = $222.75

However – in real life ……

- Every airline has it’s own database
- The database captures all relevant data and figure
- We will use those data for historical calculations

US major carriers' unit revenues and expenses by different fleet types (for reference only)

Aircraft	ASM (million)	RPM (million)	CASM*
A300-600	10,239.113	8,056.686	7.5
A319	28,714.442	23,080.522	7.2
A320	43,670.093	36,306.733	6.27
A321	7,785.539	6,582.864	5.26
A330	24,786.437	20,929.953	4.65
B717-200	12,841.303	9,581.749	7.58
B737-200	15.110	9.889	N/A
B737-300	53,346.328	39,261.932	7.45
B737-400	12,301.534	9,226.063	7.7
B737-500	14,931.963	11,715.791	8.2
B737-700	83,872.844	62,853.756	4.9
B737-800/900	71,276.880	57,523.046	5.46
B747-200	917.667	664.746	9.16
B747-400	40,405.191	33,848.281	5.61

(Con't)

B757-200	139,883.555	116,023.006	5.95
B757-300	14,372.536	12,175.073	5.11
B767-200	12,786.649	10,378.550	6.12
B767-300	78,611.122	64,333.916	5.79
B767-400	19,667.409	16,325.545	4.97
B777	77,297.169	63,606.785	6.3
DC-10-30	524.388	308.648	6.8
DC-9	10,417.344	8,007.598	10.2
EMB-190	333.023	250.417	9.70
L-1011-500	1,242.207	704.213	7.70
MD-80	71,094.571	57,170.452	7.31

* CASM = Total of type × aircraft operating cost / Total of type × aircraft ASM.

Source: The Airline Monitor, August 2008 & Back Aviation Solutions, Form41 iNET.

Domestic operations – key performance indicators for major US carriers (for reference only)

Carrier	ASM (million)	RPM (million)	RASM (cents)	CASM (cents)	Yield (cents)
Airtran	23,814.456	18,784.437	10.13	8.00	12.85
Alaska	21,815.762	16,742.678	10.80	7.76	14.07
American	101,855.071	83,313.426	10.89	8.22	13.32
Continental	52,987.792	44,215.689	10.95	7.79	13.12
Delta	128,976.090	105,697.565	10.73	7.17	13.10
JetBlue	32,435.674	26,069.180	9.45	6.69	11.76
Southwest	103,486.264	73,639.652	9.93	6.48	13.96
United	135,859.306	110,061.748	10.90	8.62	13.45
US Airways	74,148.295	60,567.144	10.77	8.51	13.18

Source: Form41 iNET.

Recapture rate

- The final spill cost needs to be adjusted by the recapture rate
- Recapture rate is the percentage of PAX that were spilled, but could be accommodated or recaptured on other flights by the same airline. In other words, if a PAX cannot get a seat on a specific flight, the airline could always offer that PAX an earlier or later flights instead.
- If that PAX accepts the offer for another flight, then this PAX is considered to be recaptured.
- Generally, airlines' recapture rate is very high (Note: depends on seasons …..).

The final expected spill costs

- If the recapture rate is 15%, which means 85% of PAX who request a reservation for a flight of that airline are denied such a request and the airline will lose those PAX to other airlines.

- Therefore, only 85% spilled PAX are the real spill costs. We can use this rate to calculate the final spill costs.

- The final ESC of a certain fleet = ESC of that fleet * (1- recapture rate)

- Therefore, the final Expected Spill Costs (ESC) of both fleets:

The final ESC of B737-800 = $2,565.33 * (1-15%) = $2,180.31
The final ESC of B757-200 = $222.75 * (1-15%) = $189.34

The fleet assignment costs

- The total fleet assignment costs = Operating costs + Expected spill costs

  - Operating cost for a B737–800 = $0.42 × 2,475 × 162 = $16,839
  - Operating cost for a B757–200 = $0.44 × 2,475 × 200 = $21,780

  - Expected spill costs for B737–800 = $2,565.33 % .85 = $2,180.31
  - Expected spill costs for B757–200 = $222.75 % .85 = $189.34

The total costs of fleet assignment (CFA) for flight 122 will be:
CFA (B737-800) = $16,839.90 + $2,180.31 = $19,020.21
CFA (B757-200) = $21,780.00 + $189.34 = $21,969.34

Fleet assignment model

$$x_{i,j} = \begin{cases} 1 \text{ if flight } i \text{ is assigned to fleet-type } j \\ 0 \text{ otherwise} \end{cases}$$

$G_{k,j}$ = integer decision variable representing number of aircraft of fleet-type j on ground at node k

- X_{ij} index i represents the flight leg, and index j represents the fleet type
- E.g. j=1 for B737-800; and j=2 for B757-200 fleets
- Thus, for flight 101,
  - $X_{101,1}$ means to assign fleet B737-800 to flight 101
  - $X_{101,2}$ means to assign fleet B757-200 to flight 101
- $G_{k,j}$ is used to address the set of constraints for aircraft balance

The objective function

- The objective function of fleet assignment is to minimize the total cost by assigning the most appropriate fleet type to all flights (42 flights):

Minimize (21485.26*$X_{101,1}$ + 22556*$X_{101,2}$ + 24222.37*$X_{102,1}$ + 23556*$X_{102,2}$ + … + 19020.21*$X_{122,1}$ + 21,969.34*$X_{122,2}$ + … + 1558.42*$X_{142,1}$ + 2006*$X_{142,2}$)

Constraints

- Flight cover
  - Ensure that each flight must be flown
- Aircraft balance
  - The set of constraints ensures that an aircraft of the right fleet type will be available at the right place at the right time.
- Fleet size
  - This set of constraints is to ensure that the number of aircraft within each fleet does not exceed the available fleet size.

Flight cover

- The first set of constraints is what is typically known as flight cover. Flight cover implies that each flight must be own. To cover a flight, the sum of all the decision variables representing that flight must add up to 1.

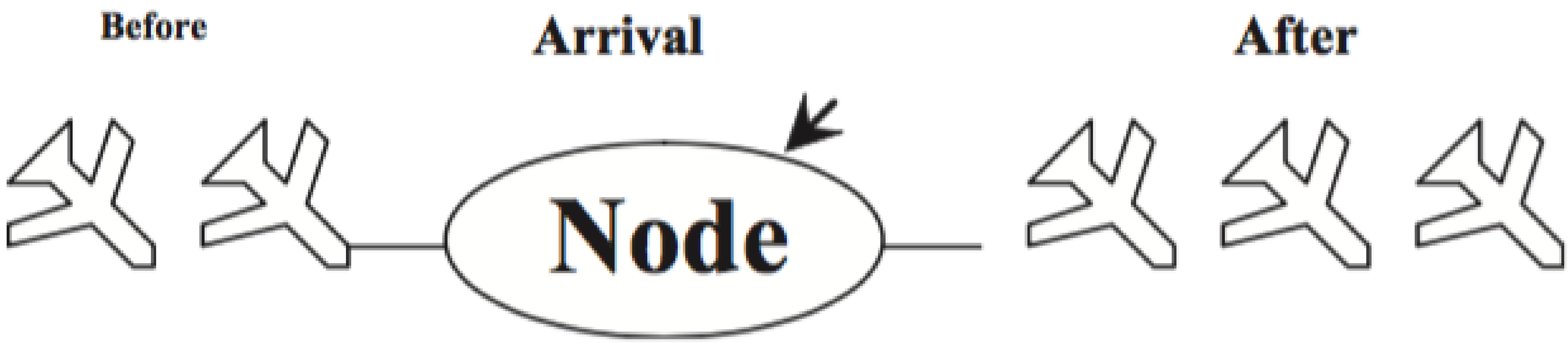
- For example:
  - $X_{101,\ 1} + X_{101,2} = 1$
  - Where 1 represents B737-800 and 2 represents B757-200

- We write similar constraints for all other 41 flights in our case study.

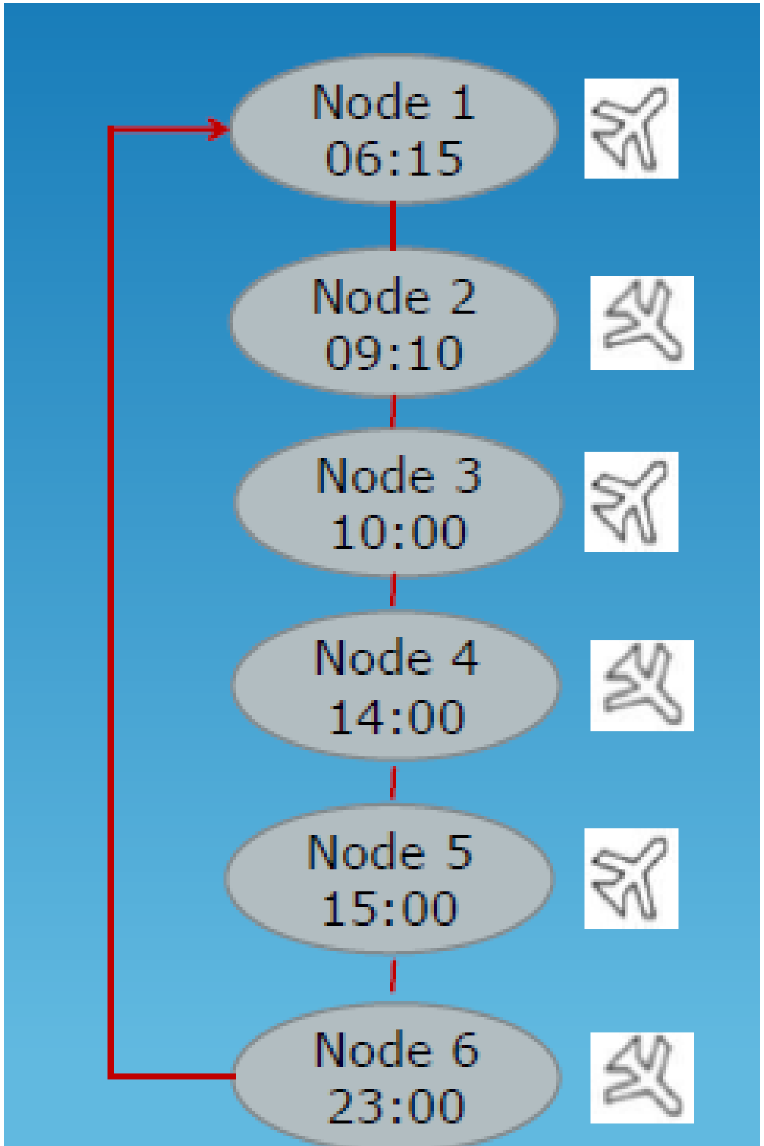
Aircraft balance or equipment continuity within the fleets.

- The balance constraint for the node:

Number of aircraft at this node = 2 (number of aircraft before this node) + 1 (one arrival) – 0 (no departure from this node) = 3

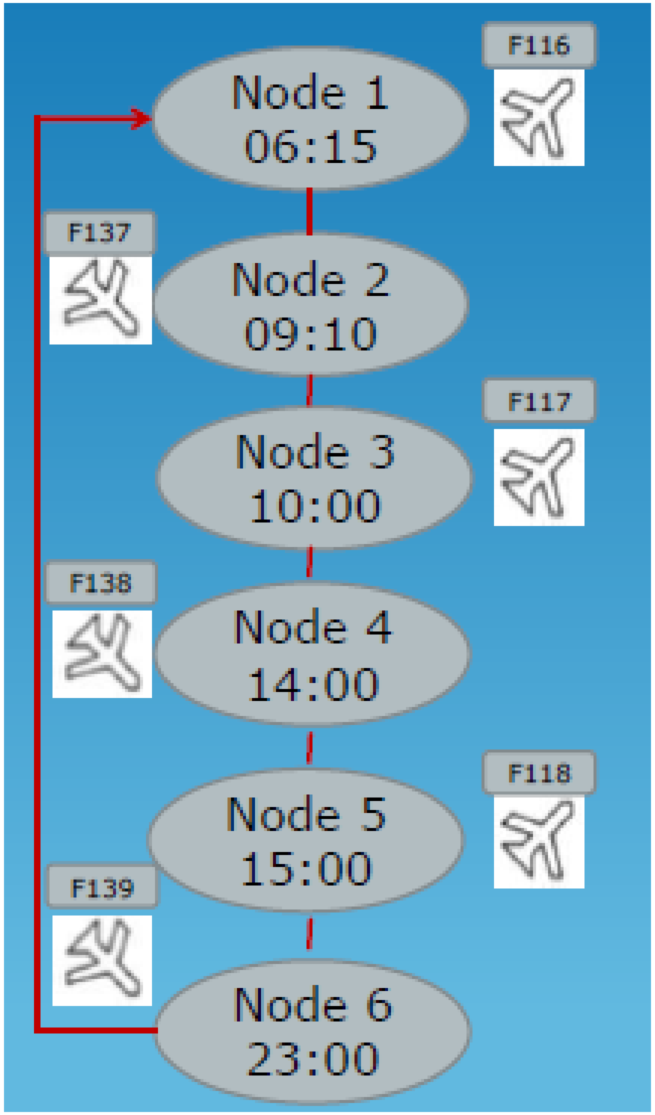


Time-space network for Boston



- If B represents Boston Airport, 1 represents B737-800 aircraft used on this route
- $G_{B6,1}$ = 1, then calculate the number of aircraft of Node 1 to Node 5. $G_{B1,1}$, $G_{B2,1}$, $G_{B3,1}$, $G_{B4,1}$ and $G_{B5,1}$

Time-space network for Boston (Con’t)



$G_{B1,1} = 1 - 1 = 0$

$G_{B2,1} = 0 + 1 = 1$

$G_{B3,1} = 1 - 1 = 0$

$G_{B4,1} = 0 + 1 = 1$

$G_{B5,1} = 1 - 1 = 0$

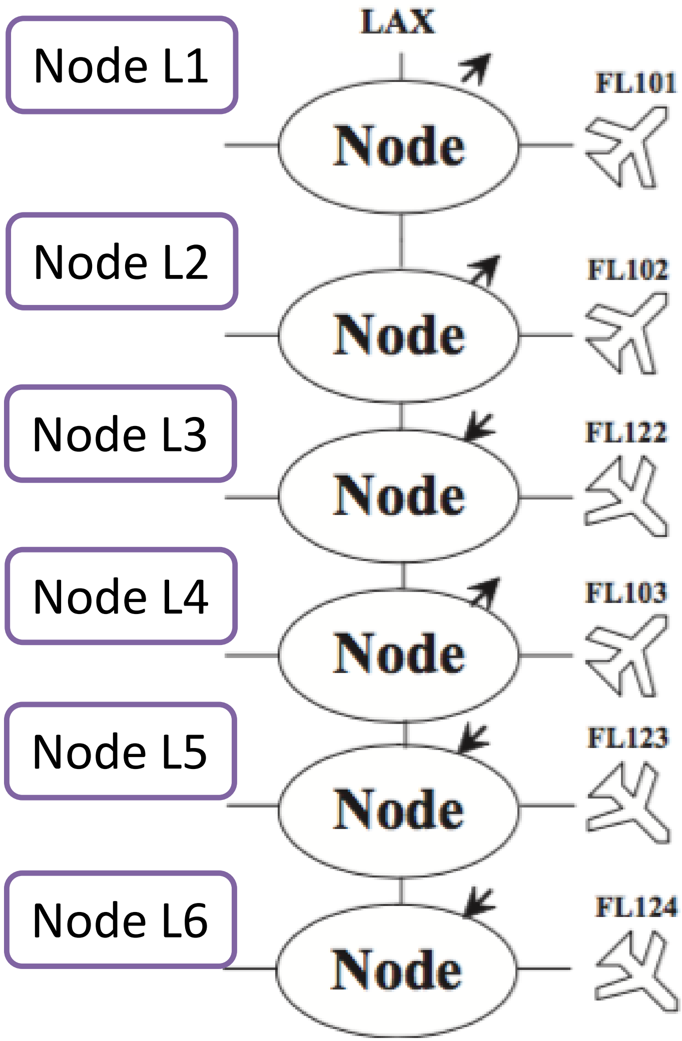
$G_{B6,1} = 0 + 1 = 1$

Arrival/departure flights for LAX

Table 4.4 Arrival/departure flights for LAX

Flight no.	Origin	Departure time	Destination	Arrival time	Duration of flight (hrs)
101	LAX	05:00	JFK	13:30	5.5
102	LAX	09:45	JFK	18:15	5.5
122	JFK	07:35	LAX	10:05	5.5
103	LAX	15:20	JFK	23:50	5.5
123	JFK	16:00	LAX	18:30	5.5
124	JFK	19:00	LAX	21:30	5.5

Aircraft balance



- For number of B737-800 aircraft
- The first node at LAX is at L1. The number of B737–800 aircraft at this node, based on the rule for balance, is basically the number of aircraft carried over from the previous day (wrap-around arc from node L6) minus one departure flight (101).

Node 1: $G_{L1,1} = G_{L6,1} - X_{101,1}$

Node 2: $G_{L2,1} = G_{L1,1} - X_{102,1}$

Aircraft balance (Con't)

At node L3, we have an arrival (flight 122), therefore:

$$G_{L3,1} = G_{L2,1} + x_{122,1}$$

Similarly, we write the other three constraints for this fleet type as follows:

$$G_{L4,1} = G_{L3,1} - x_{103,1}$$

$$G_{L5,1} = G_{L4,1} + x_{123,1}$$

$$G_{L6,1} = G_{L5,1} + x_{124,1}$$

Aircraft balance (Con't)

- The constraints for the B757–200 fleet are similar to the B737–800 as follows:
- There are 42 flights in our Ultimate Air case study. Each flight has a departure and an arrival. We have two fleet types. Therefore, the total number of constraints for aircraft balance is 168 (42 × 2 × 2).

$$G_{L1,2} = G_{L6,2} - x_{101,2}$$

$$G_{L2,2} = G_{L1,2} - x_{102,2}$$

$$G_{L3,2} = G_{L2,2} + x_{122,2}$$

$$G_{L4,2} = G_{L3,2} - x_{103,2}$$

$$G_{L5,2} = G_{L4,2} + x_{123,2}$$

$$G_{L6,2} = G_{L5,2} + x_{124,2}$$

Fleet Size

- This set of constraints is adopted to ensure that the number of aircraft within each fleet does not exceed the available fleet size.
- To address this, we must count the number of aircraft that are grounded overnight for that fleet type at different airports.
- Referring to Figure 4.4, the last node, L6 (originating node for wrap- around arc), represents the total number of aircraft in LAX at the end of the day.
- The total number of B737–800 aircraft in our network is therefore:

$$G_{L6,1} + G_{S6,1} + G_{B6,1} + G_{O6,1} + G_{A6,1} + G_{I6,1} + G_{M6,1} + G_{J42,1}$$

- Los Angeles (LAX); San Francisco (SFO); Boston (BOS); Chicago (ORD); Atlanta (ATL); Washington DC(IAD); Miami (MIA); New York (JFK)

Fleet Size (Con't)

- In our case study, Ultimate Air, assume that we have nine and six aircraft in our B737–800 and B757–200 fleets, respectively.

- Since there are only two fleet types, there are only two constraints in this set

$$G_{L6,1} + G_{S6,1} + G_{B6,1} + G_{O6,1} + G_{A6,1} + G_{I6,1} + G_{M6,1} + G_{J42,1} \leq 9$$

$$G_{L6,2} + G_{S6,2} + G_{B6,2} + G_{O6,2} + G_{A6,2} + G_{I6,2} + G_{M6,2} + G_{J42,2} \leq 6$$

Solution

- The linear integer program for fleet assignment for Ultimate Air has 252 (84 binary and 168 integer) variables and 212 constraints. Using an optimization software, the solution to this problem generates a minimum daily cost of fleet assignment of $410,612.57.

Flight no.	Origin	Destination	Fleet type
101	LAX	JFK	737-800
104	SFO	JFK	737-800
116	BOS	JFK	737-800
140	JFK	IAD	737-800
125	JFK	SFO	757-200
107	ORD	JFK	737-800
122	JFK	LAX	737-800
137	JFK	BOS	737-800
110	ATL	JFK	757-200
119	IAD	JFK	737-800
113	MIA	JFK	757-200

Solution (Con't)

131	JFK	ATL	757-200
102	LAX	JFK	737-800
105	SFO	JFK	757-200
117	BOS	JFK	737-800
128	JFK	ORD	737-800
134	JFK	MIA	737-800
141	JFK	IAD	737-800
108	ORD	JFK	737-800
138	JFK	BOS	757-200
111	ATL	JFK	757-200
120	IAD	JFK	737-800
114	MIA	JFK	757-200
132	JFK	ATL	737-800
118	BOS	JFK	757-200
129	JFK	ORD	737-800
135	JFK	MIA	757-200

Solution (Con't)

Flight no.	Origin	Destination	Fleet type
142	JFK	IAD	737-800
103	LAX	JFK	737-800
106	SFO	JFK	737-800
126	JFK	SFO	737-800
123	JFK	LAX	737-800
109	ORD	JFK	737-800
112	ATL	JFK	737-800
133	JFK	ATL	757-200
136	JFK	MIA	757-200
115	MIA	JFK	737-800
121	IAD	JFK	737-800
124	JFK	LAX	737-800
127	JFK	SFO	737-800
130	JFK	ORD	737-800
139	JFK	BOS	737-800

Scenario Analysis : Case 1

- It may be of interest to us to see what is the minimum number of aircraft to cover all flights.
- In this case, the objective function is modified to minimize the total number of aircraft.
- Therefore, the fleet size constraints are deleted from the set of constraints and become the objective function as follows:
- Results: 9 are 737– 800 and 4 are 757–200

$$Min\ G_{L6,1} + G_{S6,1} + G_{B6,1} + G_{O6,1} + G_{A6,1} + G_{I6,1} + G_{M6,1} + G_{J42,1} +$$
$$G_{L6,2} + G_{S6,2} + G_{B6,2} + G_{O6,2} + G_{A6,2} + G_{I6,2} + G_{M6,2} + G_{J42,2}$$

Scenario Analysis : Case 2

- We evaluate various combinations of the two fleets.
- We now change this combination to see its impact on total daily cost.

Number of B737-800 aircraft	Number of B757-200 aircraft	Total daily cost
8	7	$411,890
6	9	$416,116
11	4	$409,362
15	0	$413,970
0	15	$446,364

Key Reference

- Bazargan, M. (2010) Airline Operations and Scheduling. 2nd edition, Ashgate
  - Chapter 4 Fleet Scheduling