香港城市大學
City University of Hong Kong

396EM Airline Operations and Scheduling / 6075MAA Airline Scheduling and Operations

Lecture 3b
Aircraft Routing

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Learning outcomes

Understand	the definition of aircraft routing
Understand	the goal of aircraft routing
Identify	the constraints of aircraft routing
Apply	the optimisation theory to solve simple airline aircraft routing problems

Aircraft Routing

- The solution obtained from the fleet assignment in the previous lecture identifies the flow of fleet through the network.
- However, it does not identify which specific aircraft from that fleet is assigned to each flight leg.
- Aircraft routing is the process of assigning each individual aircraft (referred to as tail number) within each fleet to each flight legs.
- The aircraft routing is also referred to as aircraft rotation, aircraft assignment or tail assignment.

Fleet assignment for Ultimate Air - 1

Flight no.	Origin	Destination	Fleet type
101	LAX	JFK	737-800
104	SFO	JFK	737-800
116	BOS	JFK	737-800
140	JFK	IAD	737-800
125	JFK	SFO	757-200
107	ORD	JFK	737-800
122	JFK	LAX	737-800
137	JFK	BOS	737-800
110	ATL	JFK	757-200
119	IAD	JFK	737-800
113	MIA	JFK	757-200
131	JFK	ATL	757-200
102	LAX	JFK	737-800
105	SFO	JFK	757-200
117	BOS	JFK	737-800

Fleet assignment for Ultimate Air - 2

128	JFK	ORD	737-800
134	JFK	MIA	737-800
141	JFK	IAD	737-800
108	ORD	JFK	737-800
138	JFK	BOS	757-200
111	ATL	JFK	757-200
120	IAD	JFK	737-800
114	MIA	JFK	757-200
132	JFK	ATL	737-800
118	BOS	JFK	757-200
129	JFK	ORD	737-800
135	JFK	MIA	757-200

Fleet assignment for Ultimate Air - 3

Flight no.	Origin	Destination	Fleet type
142	JFK	IAD	737-800
103	LAX	JFK	737-800
106	SFO	JFK	737-800
126	JFK	SFO	737-800
123	JFK	LAX	737-800
109	ORD	JFK	737-800
112	ATL	JFK	737-800
133	JFK	ATL	757-200
136	JFK	MIA	757-200
115	MIA	JFK	737-800
121	IAD	JFK	737-800
124	JFK	LAX	737-800
127	JFK	SFO	737-800
130	JFK	ORD	737-800
139	JFK	BOS	737-800

B737-800 Fleet Assignment - 1

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet Type
101	LAX	5:00	JFK	13:30	5.5	737-800
104	SFO	5:05	JFK	13:35	5.5	737-800
116	BOS	6:15	JFK	7:45	1.5	737-800
140	JFK	6:20	IAD	7:20	1	737-800
107	ORD	7:30	JFK	10:30	2	737-800
122	JFK	7:35	LAX	10:05	5.5	737-800
137	JFK	7:40	BOS	9:10	1.5	737-800
119	IAD	8:15	JFK	9:15	1	737-800
102	LAX	9:45	JFK	18:15	5.5	737-800
117	BOS	10:00	JFK	11:30	1.5	737-800
128	JFK	10:05	ORD	11:05	2	737-800
134	JFK	10:35	MIA	13:35	3	737-800
141	JFK	12:00	IAD	13:00	1	737-800
108	ORD	12:20	JFK	15:20	2	737-800
120	IAD	14:25	JFK	15:25	1	737-800

B737-800 Fleet Assignment - 2

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet Type
132	JFK	14:35	ATL	17:35	2.5	737-800
129	JFK	15:05	ORD	16:05	2	737-800
142	JFK	15:15	IAD	16:15	1	737-800
103	LAX	15:20	JFK	23:50	5.5	737-800
106	SFO	15:25	JFK	23:55	5.5	737-800
126	JFK	15:30	SFO	18:00	5.5	737-800
123	JFK	16:00	LAX	18:30	5.5	737-800
109	ORD	17:10	JFK	20:10	2	737-800
112	ATL	18:00	JFK	20:30	2.5	737-800
115	MIA	18:15	JFK	21:15	3	737-800
121	IAD	18:30	JFK	19:30	1	737-800
124	JFK	19:00	LAX	21:30	5.5	737-800
127	JFK	20:00	SFO	22:30	5.5	737-800
130	JFK	21:00	ORD	22:00	2	737-800
139	JFK	21:30	BOS	23:00	1.5	737-800

B757-200 Fleet Assignment

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet type
125	JFK	7:25	SFO	9:55	5.5	757-200
110	ATL	8:10	JFK	10:40	2.5	757-200
113	MIA	9:10	JFK	12:10	3	757-200
131	JFK	9:30	ATL	12:00	2.5	757-200
105	SFO	9:50	JFK	18:20	5.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
111	ATL	13:10	JFK	15:40	2.5	757-200
114	MIA	14:30	JFK	17:30	3	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
135	JFK	15:10	MIA	18:10	3	757-200
133	JFK	18:05	ATL	20:35	2.5	757-200
136	JFK	18:10	MIA	21:10	3	757-200

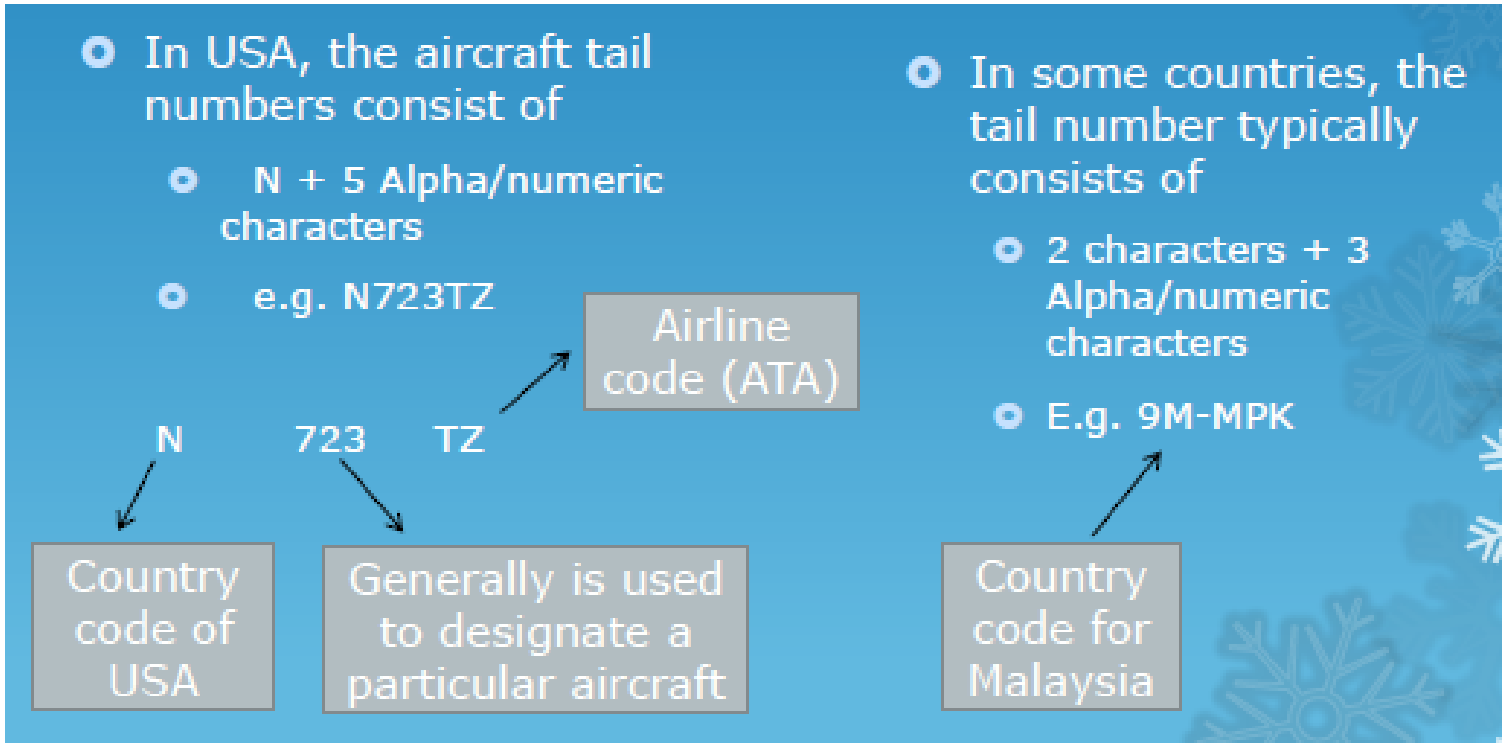
Goal of Aircraft Routing

- After obtain the solutions of the fleet assignment, a summary table of each fleet type can be generated for aircraft routing

- To assign routes to individual aircraft within a specific fleet type in order to maximise the revenue or minimise operating costs

Tail Number

- Aircraft are normally distinguished by their tail registration numbers. A tail number is a unique serial number assigned to each aircraft for each airline in each country.



Tail Number (Con't)

- Examples of the country code tail numbers:
  - HKG (e.g. B-HLL)
  - PRC (e.g. B-222)
  - UK (e.g. G-AACI)

Aircraft routing problem

To maximise the revenue or minimise operating cost with the following:

- Flight coverage: each flight leg must be covered by only one aircraft.
- Aircraft load balance: the aircraft must have balanced utilization loads.
- Maintenance requirements: not all the airports that an airline flies to have the capability to perform maintenance checks on all fleet types. The airlines normally have maintenance bases, typically at their hubs, for different fleet types. The maintenance consideration is to ensure that the aircraft are own through the network in a manner that allows them to receive the required maintenance checks at the right time and at the right base.

The rationale of the mathematical approach adopted for aircraft routing

- Adopt a set-partitioning formulation to determine the daily routing for each aircraft.
- In this approach, all possible valid aircraft routings are generated
- Identify the best routes that cover all flights while meeting maintenance requirements (opportunities) and turn-around time, routing cycles and etc.

Impact factors for aircraft routing

- Maintenance requirements
- Maintenance routing
- Routing cycles

Maintenance requirements

- the FAA mandates that the airlines perform four types of aircraft maintenance, commonly referred to as A-, B-, C- and D-checks.
- A-checks approximately every 60 flight hours.
- B-checks involve a thorough visual inspection and lubricating of all moving parts
- C- and D-checks involve taking the aircraft out of service, and are performed every one to four years.

Four Types of Aircraft Maintenance

Maintenance check	What inspection should be involved	FAA mandates airline should perform	Time required to perform a check
A check	A visual inspection of major systems	About every 60 flight hours or 48 operating days depending on aircraft utilisation	3-10 hours or 20-50 man-hours (performed between 10pm-8am)
B check	A thorough visual inspection and lubricating of all moving parts	Every 300 – 600 hours of flight (4-6 months)	1-3 days or 150 man-hours
C check	requiring a large majority of the aircraft's components to be inspected	Every 20-24 months;	1-2 weeks or 6000 man-hours
D check	"heavy maintenance visit" (HMV).	Every 6 years	Up to 2 months or 50,000 man-hours

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Adopt Set Partitioning approach to sort out the daily aircraft routing

- A valid routing needs to include the turn-around time.
- The turn-around time is the minimum time needed for an aircraft from the time it lands until it is ready to depart again.
- The time includes the taxing into the gate, unloading passengers and baggage, cleaning, inspections, boarding new passengers, loading new baggage, and etc.
- Generally, the turnaround time varies from 20 minutes to several hours depending the types of airlines and types of aircrafts

Example: Valid Routings

- In the Ultimate Air example, we assume that the turn-around time is 45 minutes.

- Flight 113 arrives at JFK at 12:10, while flight 138 departs JFK at 12:30. The turn-around time is 20 minutes, which is less than our minimum of 45 minutes.

- (see next slide)

Example for turnaround time issue

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet type
125	JFK	7:25	SFO	9:55	5.5	757-200
110	ATL	8:10	JFK	10:40	2.5	757-200
113	MIA	9:10	JFK	12:10	3	757-200
131	JFK	9:30	ATL	12:00	2.5	757-200
105	SFO	9:50	JFK	18:20	5.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
111	ATL	13:10	JFK	15:40	2.5	757-200
114	MIA	14:30	JFK	17:30	3	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
135	JFK	15:10	MIA	18:10	3	757-200
133	JFK	18:05	ATL	20:35	2.5	757-200
136	JFK	18:10	MIA	21:10	3	757-200

Routing cycles (“Loop”)

- A closed cycle is when an aircraft starts from a city and at the end of the cycle (e.g. one day, three days or a week), ends up at the same city to start another cycle.

- This requirement is included to better present the process of aircraft routing by reducing the number of potential routings.

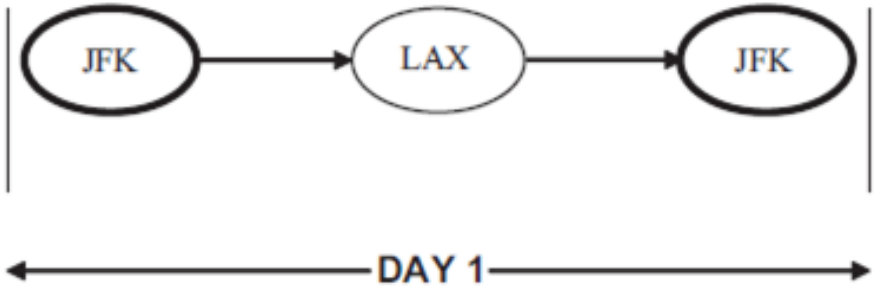
- In practice (closed cycles are not typically a requirement for airlines), airlines usually develop monthly aircraft routing with no closed cycles.

A sample of one-day routing

- The aircraft stays at JFK every night and repeats the cycle every day.
- This provides the chance for the aircraft to do the maintenance every night, but not necessary.

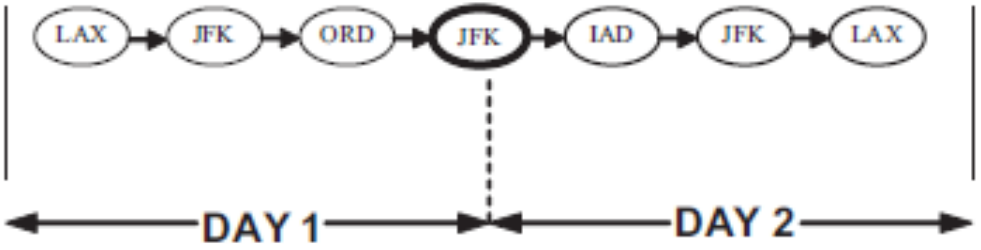
B737-800 on-day routing

Flight No.	Origin	Departure Time	Destination	Arrival Time	(Hrs.)	Fleet Type
DAY 1						
122	JFK	7:35	LAX	10:05	5.5	737-800
103	LAX	15:20	JFK	23:50	5.5	737-800



A two-day routing of B737-800 – start from LAX, and spend 1st night at JFK

Flight No.	Origin	Departure Time	Destination	Arrival Time	Flight Hrs	Fleet Type
			DAY 1			
101	LAX	5:00	JFK	13:30	5.5	737-800
129	JFK	15:05	ORD	16:05	2	737-800
109	ORD	17:10	JFK	20:10	2	737-800
			DAY 2			
140	JFK	6:20	IAD	7:20	1	737-800
120	IAD	14:25	JFK	15:25	1	737-800
127	JFK	19:00	LAX	21:30	5.5	737-800



A three-day routing diagram

- An airline can generate a weekly routings based on the mentioned 1-3 days routings with a maintenance opportunity every three days

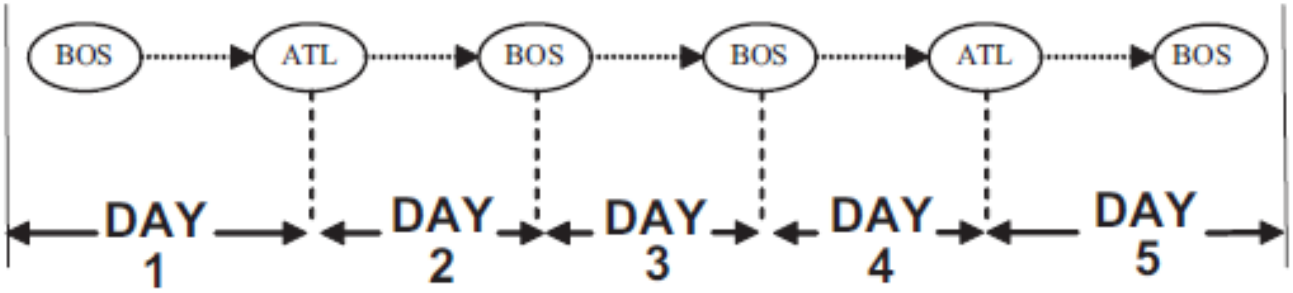
Flight No.	Origin	Departure Time	Destination	Arrival Time	Flight Hrs	Fleet Type
			DAY 1			
107	ORD	7:30	JFK	10:30	2	737-800
141	JFK	12:00	IAD	13:00	1	737-800
120	IAD	14:25	JFK	15:25	1	737-800
124	JFK	19:00	LAX	21:30	5.5	737-800
			DAY 2			
101	LAX	5:00	JFK	13:30	5.5	737-800
129	JFK	15:05	ORD	16:05	2	737-800
109	ORD	17:10	JFK	20:10	2	737-800
			DAY 3			
140	JFK	6:20	IAD	7:20	1	737-800
119	IAD	8:15	JFK	9:15	1	737-800
141	JFK	12:00	IAD	13:00	1	737-800
120	IAD	14:25	JFK	15:25	1	737-800
130	JFK	21:00	ORD	22:00	2	737-800

A 5-day routing without the opportunity for overnight maintenance at the JFK hub

Flight No.	Origin	Departure Time	Destination	Arrival Time	Flight Hrs	Fleet Type
			DAY 1			
116	BOS	6:15	JFK	7:45	1.5	757-200
131	JFK	9:30	ATL	12:00	2.5	757-200
111	ATL	13:10	JFK	15:40	2.5	757-200
133	JFK	18:05	ATL	20:35	2.5	757-200
			DAY 2			
110	ATL	8:10	JFK	10:40	2.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
139	JFK	21:30	BOS	23:00	1.5	757-200
			DAY 3			
116	BOS	6:15	JFK	7:45	1.5	757-200
131	JFK	9:30	ATL	12:00	2.5	757-200
111	ATL	13:10	JFK	15:40	2.5	757-200
139	JFK	21:30	BOS	23:00	1.5	757-200

Day 4 & 5 of the 5-day routing (Con't)

Flight No.	Origin	Departure Time	Destination	Arrival Time	Flight Hrs	Fleet Type
			DAY 4			
117	BOS	10:00	JFK	11:30	1.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
133	JFK	18:05	ATL	20:35	2.5	757-200
			DAY 5			
110	ATL	8:10	JFK	10:40	2.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
136	JFK	21:30	BOS	23:00	1.5	757-200



The programming logic of aircraft routing

- Seldom done manually NOW

- Use a featuring program to work out the possible combination

The programming logic of aircraft routing (Con't)

- Obtain the flights & fleet information with flight numbers, departure & arrival cities, departure & arrival times for a set of flights assigned to a specific fleet

- Create all possible valid one-day routings incorporating turnaround times

- Based on this one-day routings to generate a valid 3-day routings, which must follow the following criteria:

  - It starts and ends at the same city
  - Each day, flights start at the city where the aircraft ended the day before
  - An overnight stay at a certain base airport (destination) e.g. JFK occurs at least once
  - Choose each element that satisfied all the above conditions to generate the potential valid 3-day routing candidates

Sample three-day routing for B757-200 fleet (455 valid three day routings)

SAMPLE	DAY 1	DAY 2	DAY 3	Utilization (hrs)
High utilization				
Routing sample #1	**FLT 131** JFK-ATL; **FLT 111** ATL-JFK; **FLT 133** JFK-ATL	**FLT 110** ATL-JFK; **FLT 138** JFK-BOS; **FLT 118** BOS-JFK; **FLT 133** JFK-ATL	**FLT 110** ATL-JFK; **FLT 138** JFK-BOS; **FLT 118** BOS-JFK	21
City-pair routing				
Routing sample #2	**FLT 110** ATL-JFK; **FLT 138** JFK-BOS; **FLT 118** BOS-JFK; **FLT 136** JFK-MIA	**FLT 113** MIA-JFK	**FLT 138** JFK-BOS; **FLT 118** BOS-JFK; **FLT 133** JFK-ATL	17
City-pair routing				
Medium utilization				
Routing sample #3	**FLT 136** JFK-MIA	**FLT 113** MIA-JFK; **FLT 133** JFK-ATL	**FLT 110** ATL-JFK; **FLT 138** JFK-BOS; **FLT 118** BOS-JFK	14
City-pair routing				
Routing sample #4	**FLT 133** JFK-ATL	**FLT 111** ATL-JFK	**FLT 131** JFK-ATL; **FLT 111** ATL-JFK	10
City-pair routing				
Low utilization				
Routing sample #5	**FLT 138** JFK-BOS; **FLT 118** BOS-JFK	**FLT 138** JFK-BOS	**FLT 118** BOS-JFK	6
City-pair routing				

- Running a program generated a total of 6,221 and 455 valid three-day routings for the 737-800 and 757-200 fleet types respectively

Sample three-day routing for B737-800 fleet

SAMPLE	DAY 1				DAY 2				DAY 3				Utilization (hrs)
High Utilization													
Routing sample #1	**FLT 122**	**FLT 103**			**FLT 122**	**FLT 103**			**FLT 122**	**FLT 103**			33
City Pair Routing	JFK-LAX	LAX-JFK			JFK-LAX	LAX-JFK			JFK-LAX	LAX-JFK			
Routing sample #2	**FLT 137**	**FLT 117**	**FLT 123**		**FLT 101**	**FLT 129**	**FLT 109**	**FLT 139**	**FLT 116**	**FLT 134**	**FLT 115**		27
City Pair Routing	JFK-BOS	BOS-JFK	JFK-LAX		LAX-JFK	JFK-ORD	ORD-JFK	JFK-BOS	BOS-JFK	JFK-MIA	MIA-JFK		
Medium Utilization													
Routing sample #3	**FLT 109**				**FLT 137**	**FLT 117**	**FLT 124**		**FLT 101**	**FLT 142**	**FLT 121**	**FLT 130**	20
City Pair Routing	ORD-JFK				JFK-BOS	BOS-JFK	JFK-LAX		LAX--JFK	JFK-IAD	IAD-JFK	JFK-ORD	
Routing sample #4	**FLT 101**	**FLT 139**			**FLT 116**				**FLT 122**				14
City Pair Routing	LAX-JFK	JFK-BOS			BOS-JFK				JFK-LAX				
Low Utilization													
Routing sample #5	**FLT 116**	**FLT 141**	**FLT 120**	**FLT 139**	**FLT 116**				**FLT 137**				8
City Pair Routing	BOS-JFK	JFK-IAD	IAD-JFK	JFK-BOS	BOS-JFK				JFK-BOS				

Mathematical Model for 757-200 Fleet

- We define the following binary decision variable to find such routings for the 757-200 fleet.

Let $$x_j = \begin{cases} 1 \text{ if route } j \text{ is selected}, & j=1,2,..,455 \\ 0 \text{ otherwise} \end{cases}$$

Objective function with different measures

- Maximise through values
  - Non-stop flights are the first choice, then connect flight remain on-board, then connect flight with deplaning but walking to their connecting flight gate, the connect flight with deplaning a remote gate.
- Minimise cost
- Maximise maintenance opportunities

Objective Function (Con't)

$$\text{Maximize} \sum_{j=1}^{455} m_j x_j$$

where:

m_j = the number of maintenance opportunities for route j. The values that m_j can take are 1, 2 and 3.

Note that as we discussed earlier, we have 455 valid routings for the flights assigned to 757-200 fleet.

Constraints: Flight Coverage

- Each routing candidate covers a certain number of flights in its three-day cycle.
- Each flight must be covered everyday. For example, sample 1 routing candidate for the 757-200 fleet in Slide p.29 covers flights 131,111 and 133 in day one.
- In day two it covers flights 110,138, 118 and 133.

Constraints: Flight Coverage (Con't)

- Searching through all the 455 routing candidates, only six candidates actually cover flight 125 in different days

Routing Candidate Variable	Day 1	Day 2	Day 3
x_1	125	105	131-111
x_2	125	105	138-118
x_3	131-111	125	105
x_4	138-118	125	105
x_5	105	131-111	125
x_6	105	138-118	125

Constraints: Flight Coverage (Con't)

- To cover flight 125 in day one, we write the following constraint: $X_1 + X_2 = 1$
  - $X_3 + X_4 = 1$ (day two)
  - $X_5 + X_6 = 1$ (day three)

- Similarly, we write the constraints for the other 11 flights.

- The total number of constraints required to cover all daily flights for the 757-200 fleet is 36 (12 flights × 3-day cycles).

Constraints: Number of Available Aircraft

- We assumed that we have six 757-200 aircraft.

$$x1 + x2 + \ldots + x455 \leq 6$$

Oops! - No Solution!

- We used an optimization software to solve this problem. The program reported that there is no feasible solution to this problem!
- The fleet-routing problem does not consider the following constraints that we have imposed on our aircraft routings.
  - A 45-minute turn-around time.
  - Three-day closed cycles, starting and ending at the same city.
- These additional constraints in the aircraft-routing problem result in an infeasible solution for our problem.

Feasible 8 aircraft solution for the 757-200 Fleet

- To search for solutions, we eliminated the constraint on the number of available aircraft to see how many aircraft would be needed to fly the proposed daily schedule of flights assigned to the 757-200 fleet.

Routing	DAY 1	DAY 2	DAY 3
1	125	105	138-118
2	110	131-111	131-111-133
3	113-135	114	136
4	131-111-136	113-136	114
5	105	138-118	125
6	114	135	113-135
7	138-118	125	105
8	133	110-133	110

Flight Schedule – Real Life Issues

- It should be noted that the airlines frequently face this problem where the existing aircraft are not enough to fly the proposed schedule.

- The main reason is that the arriving and departing flights in the proposed schedule are not synchronized (Note: This is a US term, the UK term – Aligned).

Check the schedule and set of constraints, see if any two flights can be paired and do the adjustment

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet type
125	JFK	7:25	SFO	9:55	5.5	757-200
105	SFO	9:50	JFK	18:20	5.5	757-200

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet type
125	JFK	07:25	SFO	09:55	5.5	757-200
105	SFO	10:50	JFK	19:20	5.5	757-200

What can we do?

- Revise the departure and arrival time
- Use a different (e.g. additional) aircraft
- Others ……

Summary: Mathematical models of aircraft routing

- This model was proposed by Kabbani and Patty (1992)

Sets

F = Set of flights
R = Set of feasible routings

Indices

j = Route index
i = Flight index

Parameters

c_j = Cost of route j
a_{ij} = 1 if flight i is covered by route j, and 0 otherwise
N = Total number of aircraft in the fleet

Decision variable

$$x_j = \begin{cases} 1 \text{ if route } j \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$$

The objective function and constraints functions

$$\textit{Minimize} \quad \sum_{j \in R} c_j x_j$$

Subject to:

$$\sum_{j \in R} a_{i,j} x_j = 1 \qquad \text{for all } i \in F$$

$$\sum_{j \in R} x_j \le N$$

$$x_j \in \{0,1\} \qquad \text{for all } j \in R$$

Ensures that each flight is covered by one and only one route

Restricts the number of selected routes to the available number of aircraft within a particular fleet type

Quick Summary

- What are the differences between fleet assignment and aircraft routing?

Fleet Assignment vs. Aircraft routing

- Fleet assignment identifies the flow of fleet through the network, but cannot tell you which specific aircraft from that fleet is assigned to each flight leg.

- Aircraft routing is the process of assigning each individual aircraft (referred to as tail number) within each fleet to flight legs.

- Thus, it also referred to as aircraft rotation, aircraft assignment or tail assignment.

Key Reference

- M. Bazargan (2010) Airline Operations and Scheduling. 2nd edition, Ashgate
  - Chapter 5 Aircraft Routing
  - Chapter 4 Fleet Assignment

- Cathay Pacific – Routes and destinations video

  https://www.youtube.com/watch?v=U4AntoPlEt4&list=RDCMUCDF__e4OyxESzW1snu1i0Bg&index=1

- Cathay Pacific – Aircraft routing video

  https://www.youtube.com/watch?v=4qtDgCjdYGU