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396EM Airline Operations and Scheduling / 6075MAA Airline Scheduling and Operations

Lecture 3c
Crew Planning

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Learning outcomes

Understand	the implementation of crew pairing and crew rostering
Formulate	the math model for crew pairing and crew rostering

Crew cost for US major carriers

- Flight-crew expenses includes per diems and other expenses incurred for hotels, parking, meals, taxi-cabs, among others, in order for an airline to maintain its crew at a city other than their home base.

Carrier	Number of flight crew[1]	Flight crew expenses[2] (000)	Crew expense/ operating expense[2] (%)
Alaska	1,455	180,845,000	5.57%
AirTran	1,632	157,383,851	6.00%
American	11,166	1,152,808,000	4.48%
Continental	4,867	623,767,000	4.05%
Delta	12,299	802,811,000	3.84%
Southwest	5,915	965,329,000	9.13%
United	6,478	757,020,000	3.44%
US Airways	5,275	482,044,882	3.39%

Source: Airline Pilot Central[1]; BackAviation Form 41 iNET.[2]

Crew Scheduling

- Unlike the fuel cost, a large portion of flight-crew expenses are controllable.

- Crew scheduling is one of the most computationally intensive combinatorial problems.

- The crew scheduling problem is typically solved in two phases, crew pairing and crew rostering.

- This is mainly because the two problems are too large to address simultaneously.

Crew Pairing

- Crew pairing is a sequence of flight legs, within the same fleet, that starts and ends at the same crew base.
- A crew base is the home station or city in which the crew actually lives.
- Large airlines typically have several crew bases. The sequence of crew pairing must satisfy many constraints such as union, government, and contractual regulations.
- A crew pairing sequence may typically span from one to five days, depending on the airline.

Two phases

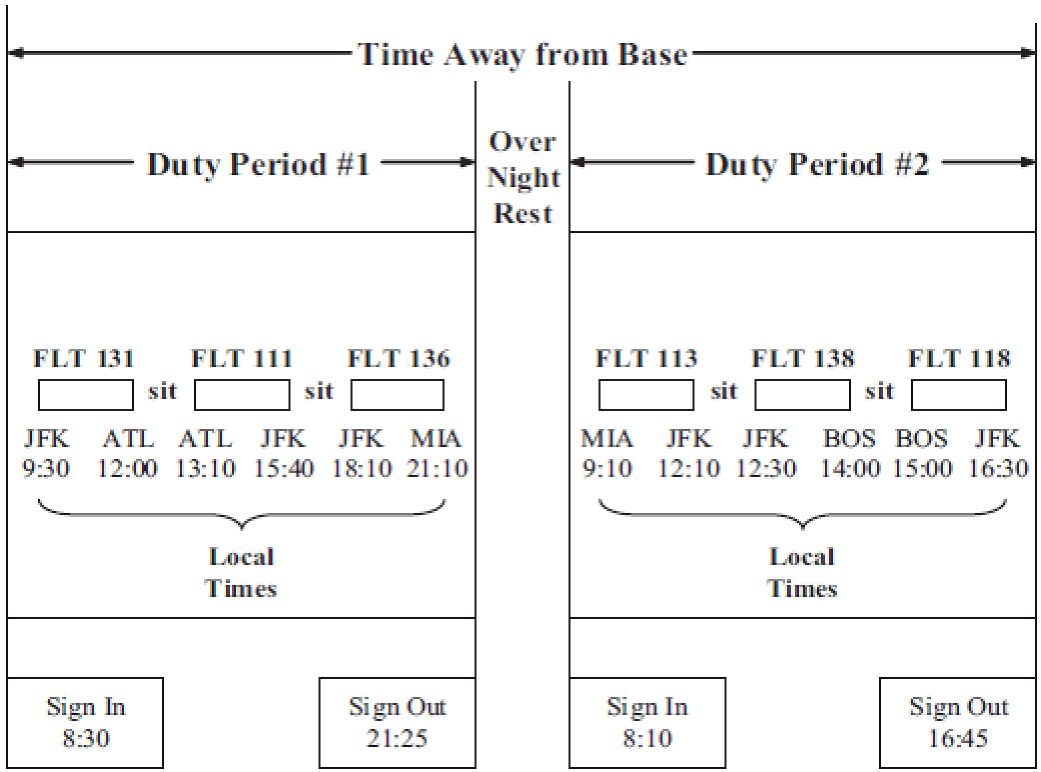
- In this phase of crew pairing, we generate pairings of flight legs that are feasible and satisfy the regulations.
- This phase does not address individual crew members.
- This phase is referred to as an impersonal phase.
- The assignment of each specific crew member to these pairings will be discussed in crew rostering.

Duty, Sit Connection & Rest

- *Duty: A working day of a crew may consist of several flight segments.*
  - In US, the length of a duty is determined by Federal Aviation Regulations (FAR), as well as by individual airline rules.
  - Under the Federal law, airline pilots cannot fly more than 8 hours in a 24-hour period. They also must be able to rest for 8 hours in that same time span.
- In Hong Kong, Civil Aviation Department has CAD 371
- *Sit connection*: A connection during duty is called a sit connection. This involves the waiting times, on the part of the crew, for changing planes onto their next leg of duty. Normally, airlines impose minimum and maximum sit connection times, typically between 10 minutes and 3 hours.
- *Rest*: A connection between two duties is referred to as rest, overnight connection or layover.

Two-day Pairing

- Two-day pairing suggests, the crew is staying overnight, away from their home base, and therefore, the airline has to pay for their per diems, transportation, accommodation, food, and so on.



Deadheading

- The airlines normally attempt to keep the crew with the same aircraft (tail number) on multiple flight legs as much as possible.
- This way, crew-related problems, such as delays and cancelled connecting flights, will be reduced.
- Delayed, cancelled connecting flights, or other difficulties in flight pairings result in deadheading.
- Deadheading happens when the crew is transported as nonrevenue passengers.

Aircraft Routing vs Crew Pairing

- The solutions for the aircraft routing and crew pairing cannot be the same.
  - Crew members need more rest. An aircraft can be utilized for 14 hours in one day, but the crew can stay with the aircraft only 8 hours.
  - Crew pairing identifies flight legs that start and end at the same crew base (i.e., only JFK to JFK in our case). This is not a constraint for the aircraft routing problem (where, for example SFO to SFO is possible) as long as it stays at a maintenance station overnight every 3–5 days.
  - Crew pairing problem does not consider turn-around times as they may just land with one aircraft and takeoff with another in a very short time.

Pairing Generators

- A pairing satisfying all the rules and regulations is called a legal pairing.

- For large airlines with many daily flights, the number of pairings generated becomes very large (billions of legal pairings!).

Crew pairing requirements for Ultimate Air

- Each duty should not exceed 8 hours of flight time.
- A maximum length of two days is allowed for a routing (i.e., two-day pairings).
- The home base for the crew is JFK.
- The minimum and maximum sit-connection times are 10 minutes and 3 hours respectively.

Computer Program Algorithm

- A similar program to route generators as in Aircraft Routing:
- Read the flight numbers, along with their departure and arrival cities and times, for a set of flights assigned to a specific fleet type (as identified by fleet-routing module).
- Create all possible one and two-day pairings
- Examine each pairing so that:
  - the pairing ends up at JFK over the routing cycle;
  - for two-day pairing, the first flight of the second day starts out at the city where it ended up the night before;
  - the duty does not exceed eight hours of flight time in any given day;
  - the sit-connection times are between the allowable minimum and maximum times.
- If a pairing satisfies all of the above conditions, it is added to a file of potential valid pairing candidates.

Results

- A total of 28 and 314 legal pairings for the 757-200 and 737-800 fleet types respectively.
- The number of crew pairing candidates is much lower than potential aircraft routings for both fleet types.
- The main reason is that for crew pairing we generated only one- and two-day pairings as opposed to three-day routings.

Decision Variables

- We define the following binary decision variable:

$$x_j = \begin{cases} 1 \text{ if pairing } j \text{ is selected,} & j = 1,2,..,28 \\ 0 \text{ otherwise} \end{cases}$$

Table 6.2: All legal crew pairings for B757-200 fleet (28 pairings)

Crew pairing index	Day-one flights	Day-two flights	Flight hours
1	125	105	11
2	131	110	5
3	131	111	5
4	131	110-138-118	8
5	133	110	5
6	133	111	5
7	133	110-138-118	8
8	135	113	6
9	135	114	6
10	135	113-138-118	9
11	136	113	6
12	136	114	6
13	136	113-138-118	9
14	138	118	3

All legal crew pairings for B757-200 fleet (Con't)

15	131-111		5
16	131-111-133	110	10
17	131-111-133	111	10
18	131-111-133	110-138-118	13
19	131-111-136	113	11
20	131-111-136	114	11
21	131-111-136	113-138-118	14
22	138-118		3
23	138-118-133	110	8
24	138-118-133	111	8
25	138-118-133	110-138-118	11
26	138-118-136	113	9
27	138-118-136	114	9
28	138-118-136	113-138-118	12

Sample one-day crew pairing for B737-800 fleet

SAMPLE	DAY 1				Crew utilization (hrs)
High utilization					
Pairing #1	FLT 140	FLT 119	FLT 128	FLT 108	6
City-pairs	JFK-IAD	IAD-JFK	JFK-ORD	ORD-JFK	
Dept-Arr times	6:20-7:20	8:15-9:15	10:05-11:05	12:20-15:20	
Low utilization					
Pairing #2	FLT 140	FLT 119			2
City-pairs	JFK-IAD	IAD-JFK			
Dept-Arr times	6:20-7:20	8:15-9:15			

Sample two-day crew pairing for B737-800 fleet

SAMPLE	DAY 1			DAY 2			Crew utilization (hrs)
High utilization							
Pairing #3	**FLT 142**	**FLT 121**	**FLT 127**	**FLT 104**	**FLT 142**	**FLT 121**	15
City pairs	JFK-IAD	IAD-JFK	JFK-SFO	SFO-JFK	JKF-IAD	IAD-JFK	
Dept-Arr times	15:15-16:15	18:30-19:30	20:00-22:30	5:05-13:35	15:15-16:15	18:30-19:30	
Medium utilization							
Pairing #4	FLT 132	FLT 112	FLT 130	FLT 107	FLT 141	FLT 120	11
City pairs	JFK-ATL	ATL-JFK	JFK-ORD	ORD-JFK	JFK-IAD	IAD-JFK	
Dept-Arr times	14:35-17:35	18:00-20:30	21:00-22:00	7:30-10:30	12:00-13:00	14:25-15:25	
Low utilization							
Pairing #5	FLT 140			FLT 119			2
City pairs	JFK-IAD			IAD-JFK			
Dept-Arr times	6:20-7:20			8:15-9:15			

Objective Function

- In our Ultimate Air example, we assume two-day pairings to be three times as costly as one-day pairings. (Crew stays away from home base for one night)

- The objective functions for our 757-200 fleet, therefore, is as follows:

$$\text{Minimize} \sum_{j=1}^{28} c_j x_j$$

where:

c_j = the cost of pairing j.

For our Ultimate Air, c_j is designated the value 1 for one-day, and 3 for two-day pairings.

Flight-Coverage Constraints for B757-200 Fleet

- Assume the crew covers each flight exactly once.
- The coverage constraint for flight 125, according to Table 6.2 (slide#16, 17) as
  - $X_1 = 1$
- This is because flight 125 only appears in crew pairing 1.
- Flight 114 appears in crew pairings 9, 12, 20, and 27.
  - $X_9 + X_{12} + X_{20} + X_{27} = 1$
- Write the flight coverage constraints for the other 10 flights with this fleet type.

Table 6.5: Solution to crew pairing for B757-200 Fleet

- Four two-day pairings were selected

Solution	DAY 1			DAY 2		
Pairing #1	**FLT 125**			**FLT 105**		
City pairs	JFK-SFO			SFO-JFK		
Dept-Arr times	7:25-9:55			9:50-18:20		
Pairing #2	**FLT 131**	**FLT 111**	**FLT 136**	**FLT 113**	**FLT 138**	**FLT 118**
City pairs	JFK-ATL	ATL-JFK	JFK-MIA	MIA-JFK	JFK-BOS	BOS-JFK
Dept-Arr times	9:30-12:00	13:10-15:40	18:10-21:10	9:10-12:10	12:30-14:00	15:00-16:30
Pairing #3	**FLT 135**			**FLT 114**		
City pairs	JFK-MIA			MIA-JFK		
Dept-Arr times	15:10-18:10			14:30-17:30		
Pairing #4	**FLT 133**			**FLT 110**		
City pairs	JFK-ATL			ATL-JFK		
Dept-Arr times	18:05-20:35			8:10-10:40		

Solution to crew pairing for B737-800

- Similarly, we can develop the mathematical model for crew pairing of 737-800 fleet

Table 6.6 Solution to crew pairing for B737-800 fleet

Solution	DAY 1			DAY 2		
Pairing #1	FLT 140	FLT 119	FLT 134	FLT 115		
City pairs	JFK-IAD	IAD-JFK	JFK-MIA	MIA-JFK		
Dept-Arr times	6:20-7:20	8:15-9:15	10:35-13:35	18:25-21:25		
Pairing #2	FLT 122			FLT 103		
City pairs	JFK-LAX			LAX-JFK		
Dept-Arr times	7:35-10:05			15:20-23:50		
Pairing #3	FLT 137	FLT 117				
City pairs	JFK-BOS	BOS-JFK				
Dept-Arr times	7:40-9:10	10:00-11:30				
Pairing #4	FLT 141	FLT 120	FLT 125	FLT 104	FLT 142	FLT 121
City pairs	JFK-IAD	IAD-JFK	JFK-SFO	SFO-JFK	JFK-IAD	IAD-JFK
Dept-Arr times	12:00-13:00	14:25-15:25	7:25-9:55	5:05-13:35	15:15-16:15	18:30-19:30
Pairing #5	FLT 132	FLT 112	FLT 130	FLT 107		
City pairs	JFK-ATL	ATL-JFK	JFK-ORD	ORD-JFK		
Dept-Arr times	14:35-17:35	18:00-20:30	21:00-22:00	7:30-10:30		
Pairing #6	FLT 129	FLT 109	FLT 139	FLT 116	FLT 128	FLT 108
City pairs	JFK-ORD	ORD-JFK	JFK-BOS	BOS-JFK	JFK-ORD	ORD-JFK
Dept-Arr times	15:05-16:05	17:10-20:10	21:30-23:00	6:15-7:45	10:05-11:05	12:20-15:20
Pairing #7	FLT 123			FLT 102		
City pairs	JFK-LAX			LAX-JFK		
Dept-Arr times	16:00-18:30			9:45-18:15		
Pairing #8	FLT 124			FLT 101		
City pairs	JFK-LAX			LAX-JFK		
Dept-Arr times	19:00-21:30			5:00-13:30		
Pairing #9	FLT 127			FLT 106		
City pairs	JFK-SFO			SFO-JFK		
Dept-Arr times	20:00-22:30			15:25-23:55		

Crew Pairings Summary

- Crew pairing mathematical model:
  - Objective Function: Minimize the total cost of flight pairings

$$\text{Min} \sum_{j \in P} c_j x_j$$

  - Constraints: guarantees each flight leg is covered only once

$$\sum_{j \in P} a_{i,j} x_j = 1 \qquad \text{for all flight legs } i \in F$$

$$b_{lower_k} \le \sum_{j \in P} h_{k,j} x_j \le b_{upper_k} \qquad \text{for all home bases } k \in K$$

  - Side Constraints: ensure selected flight pairings stay within the available number of crew members at each home base.

Crew Pairings Summary (Con't)

- Sets
  - F = Set of flights
  - P = Set of feasible pairings
  - K = Set of crew home-base cities
- Indices
  - j = Pairing index
  - i = Flight index
  - k = Crew home-base index

Crew Pairings Summary (Con't)

- Parameters = Cost of crew pairing j

$$a_{i,j} = \begin{cases} 1 \text{ if flight } i \text{ is covered by pairing } j \\ 0 \text{ otherwise} \end{cases}$$

$$h_{k,j} = \begin{cases} 1 \text{ if home base city (starting and ending flight) for pairing } j \text{ is city } k \\ 0 \text{ otherwise} \end{cases}$$

$$b_{lower_k} = \text{ minimum number of crew to be used at home base city } k$$

$$b_{upper_k} = \text{ maximum number of crew to be used at home base city } k$$

- Decision Variable

$$x_j = \begin{cases} 1 \text{ if pairing } j \text{ is part of the solution} \\ 0 \text{ otherwise} \end{cases}$$

Crew Rostering

- Crew rostering is the process of assigning individual crew members to crew pairings, usually on a monthly basis.
- Since the rules and regulations vary among the airlines, the crew rostering process is also diverse. Some of these methods include
  - assigning high priority employees to high priority pairings
  - developing monthly rosters for individual crew members based on their requests
  - developing monthly rosters for each day of the month without considering the crew requests

Cockpit Aircrew Members

- Assigning cockpit aircrew members (Captain and First Officer) and cabin aircrew (Flight Attendants) are typically different.

- The cockpit aircrew members usually have the required licenses/type ratings to fly only a specific fleet of aircraft, while cabin aircrew members can be assigned to multiple fleet types.

Simplification for Our Case

- For Ultimate Air, we attempt to develop anonymous rosters on which its employees can bid.

- In an effort to keep the rostering problem to a manageable size, we will develop the rosters on a weekly basis, instead of monthly rosters which are more common among airlines

Assumption for the Ultimate Air crew rosters

- At least one day off between pairings;
- Two pairings per week;
- balanced workload among all rosters – a work week of 20 flight hours is desirable.

Possible weekly crew roster combinations for Ultimate Air

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
✔			✔			
✔				✔		
	✔			✔		
	✔				✔	
		✔			✔	
		✔				✔
			✔			✔

Explanation

- Each (✓) symbol represents a pairing.
- Each pairing spans a two-day period.
- If a crew is assigned to a pairing on Monday, then this crew member will be flying both on Monday and Tuesday.
- Since we require at least one day rest between pairings, this crew member cannot fly on Wednesday, but can fly on Thursday, Friday, Saturday or Sunday.

Crew Rosters for B757 Fleet

- Table 6.5 (Slide #22) presented the solution to our crew pairing phase for the 757-200 fleet.

- Let us call these four pairings P1, P2, P3, and P4.

- The next slide presents three sample valid rosters with corresponding total weekly flight hours.

Three sample rosters for B757-200 Fleet (112 rosters in total)

Sample Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight hrs
1	P1			P2				25
2			P3			P1		17
3		P4			P2			19

$$x_j = \begin{cases} 1 \text{ if roster } j \text{ is selected}, & j=1,2,..,112 \\ 0 \text{ otherwise} \end{cases}$$

Objective Function

- A major goal of Ultimate Air is to create balanced rosters around 20 weekly flights hours.
- The objective function is to minimize the total deviations of the rosters' weekly flight hours from target of 20 flight hours and represented as:

$$\textit{Minimize} \quad \sum_{j=1}^{112} \left| h_j - 20 \right| \cdot x_j$$

where:

h_j = the total weekly flight hours for roster j.

Pairing Coverage Constraints

- A simple program similar to Aircraft Routing can search through our 112 candidates to identify which ones cover which pairings, and on what days.

- We have four pairings that need to fly every day of the week, which makes a total of (4 × 7) 28 constraints as follows:

$$\sum_{j=1}^{112} a_{i,j} x_j = 1 \qquad \textit{For all } i = 1, 2, .., 28$$

Pairing Coverage Constraints

- In this set of constraints, index i represents a specific pairing in a given day. As an example, the number 1 represents P1 on Monday, while 2 stands for P2 on Monday…., and 28 is P4 on Sunday. The parameter a_{ij} is defined as:

$$a_{i,j} = \begin{cases} 1 \text{ if roster } j \text{ covers pairing } i \\ 0 \text{ otherwise} \end{cases}$$

Solution to crew rosters for B757-200 fleet

Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight Hours
1	0	P1	0	0	P3	0	0	17
2	0	0	P1	0	0	P3	0	17
3	P1	0	0	P4	0	0	0	16
4	0	0	0	P2	0	0	P3	20
5	P2	0	0	0	P4	0	0	19
6	0	P2	0	0	0	P4	0	19
7	0	0	P2	0	0	0	P4	19
8	0	P3	0	0	P1	0	0	17
9	0	0	P3	0	0	0	P1	17
10	P3	0	0	0	P2	0	0	20
11	0	0	0	P3	0	0	P2	20
12	P4	0	0	P1	0	0	0	16
13	0	P4	0	0	0	P1	0	16
14	0	0	P4	0	0	P2	0	19

Solution to crew rosters for B757-200 fleet (Summary)

- 14 disjointed (non-overlapping) rosters, each covering two pairings per day.
- Each pairing covered exactly once every day.
- For a more balanced flight hours, one possible approach is to rotate the rosters every week among the crew members.
- As a result, 14 captains and 14 first officers for 757-200 is needed
- Based on rules and regulations, rosters can be assigned to any individual crew member.

Solution to crew rosters for B737-800 fleet

- Similar approach is adopted for B737-800.
- Nine pairings => got 567 valid rosters and 63 constraints. (9 pairings x 7 days/week)
- Solution generates total of 43 hours deviation for 32 rosters.

Solution to crew rosters for B737-800 fleet (Con’t)

Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight Hours
1	0	0	P1	0	0	0	P7	19
2	P1	0	0	0	P9	0	0	19
3	0	P1	0	0	0	P9	0	19
4	0	0	0	P1	0	0	P9	19
5	P2	0	0	P2	0	0	0	22
6	0	P2	0	0	0	P5	0	20
7	0	0	P2	0	0	0	P5	20
8	P3	0	0	P4	0	0	0	18
9	0	P3	0	0	P4	0	0	18
10	0	0	P3	0	0	P4	0	18
11	0	0	0	P3	0	0	P4	18
12	P4	0	0	0	P3	0	0	18
13	0	P4	0	0	0	P3	0	18
14	0	0	P4	0	0	0	P3	18
15	P5	0	0	0	P6	0	0	20
16	0	0	P5	0	0	0	P6	20
17	0	P5	0	0	0	P8	0	20
18	0	0	0	P5	0	0	P8	20
19	0	0	0	P6	0	0	P1	19

Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight Hours
20	P6	0	0	P8	0	0	0	22
21	0	P6	0	0	P8	0	0	22
22	0	0	P6	0	0	0	P9	22
23	0	0	0	P7	0	0	P2	22
24	0	P7	0	0	0	P6	0	22
25	0	0	P7	0	0	P7	0	22
26	P7	0	0	P9	0	0	0	22
27	P8	0	0	0	P1	0	0	19
28	0	P8	0	0	P2	0	0	22
29	0	0	P8	0	0	P2	0	22
30	0	0	P9	0	0	P1	0	19
31	0	P9	0	0	P5	0	0	20
32	P9	0	0	0	P7	0	0	22

Crew Rostering Summary

- Crew rostering mathematical model:
  - Objective Function: Minimize the total sum of deviations

$$\text{Minimize} \sum_{j \in R} c_j x_j$$

  - Constraints: guarantees each flight pairing in each day is covered only once

$$\sum_{j \in R} a_{i,j} x_j = 1 \qquad \text{For all} \qquad i \in P$$

Crew Rostering Summary (Con't)

Sets

- P = Set of pairings over all days of the roster period
- R = Set of valid rosters

Indices

- j = Roster index
- i = Pairing index

Crew Rostering Summary (Con't)

- Parameters

c_j = Deviation of roster *i* flight time from a target value

$$a_{i,j} = \begin{cases} 1 \text{ if pairing } i \text{ is covered by roster } j \\ 0 \text{ otherwise} \end{cases}$$

- Decision Variable

$$x_j = \begin{cases} 1 \text{ if roster } j \text{ is part of the solution} \\ 0 \text{ otherwise} \end{cases}$$

Key Reference

- Bazargan, M. (2010) Airline Operations and Scheduling. 2nd edition, Ashgate
  - Chapter 6 Crew Scheduling

- Crew Roster and Planning example (AerOPS system)
  https://www.youtube.com/watch?v=d4OhcQVVmzQ

- Crew Management (Sabre)
  https://www.youtube.com/watch?v=Cb2B4UkGWvE