



香港城市大學
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396EM Airline Operations and Scheduling/ 6075MAA Airline Scheduling and Operations

Lecture 4b Gate Assignment



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Understand the principles of Gate assignment



Formulate the Mathematical model of Gate assignment in passengers and baggage handling.

- ▶ Assigning arriving flights to airport gates have a major impact on maintaining **efficiency of flight schedules and passenger satisfaction**.
- ▶ Factors affect the gate assignment: Aircraft size, passenger walking distance, baggage transfer, ramp congestion, aircraft rotation and aircraft service requirements (Gu and Chung 1999)

Gate assignment handling

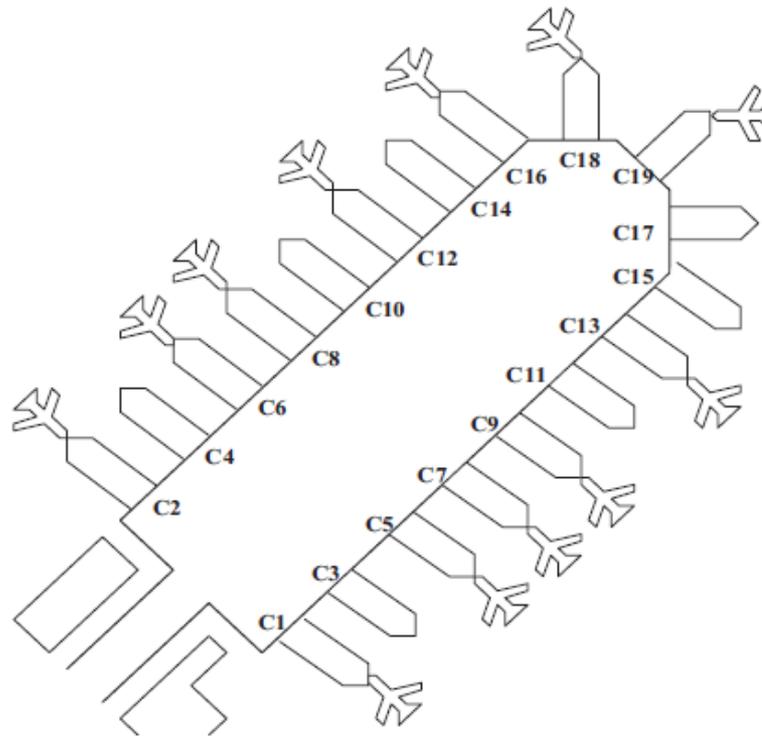
► Usually there are 3 levels

- Ground controllers use the flight schedule to examine the capacity of gates to accommodate the flights.
- Develop daily plans before the actual day of operation
- Daily plan are updated and revised due to irregular conditions (delays, bad weather, mechanical failure ..etc)

Case Study

(Minimise Passenger walking distance)

- C Concourse at San Francisco (SFO)
- Total 19 gates (C1-C19)
- 12 aircraft already at the gates for departure



Case Study (Con't)

- ▶ 7 flights will be arrived and assigned to the remaining gates
- ▶ F1-F7 refer to the flights
- ▶ Passengers will connect to other departing flights from these 7 flights
- ▶ Number of Passengers flow is shown below

Flight	Departing gates																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
F1	5	5	10	8	15	8	2	10	8	20	5	4	0	9	3	4	1	2	1
F2	5	2	1	4	19	9	4	2	3	2	27	3	8	4	0	2	1	7	2
F3	10	0	4	9	13	4	4	4	3	5	5	8	4	9	11	7	9	4	4
F4	4	8	5	4	10	4	1	0	0	2	4	19	1	2	4	5	5	8	2
F5	4	11	9	9	6	3	1	4	4	2	1	0	3	5	1	2	2	3	4
F6	1	2	42	5	2	7	6	2	4	7	2	3	6	4	10	2	1	0	0
F7	3	3	2	5	9	13	11	2	2	3	7	22	4	0	1	1	2	2	9

Case Study (Con't)

- Distance in yards between the gates are shown below

Gates	Departing Gates																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
3	10	40	-	30	10	40	20	50	30	60	40	70	50	80	60	90	70	90	80
4	40	10	30	-	40	10	50	20	60	30	70	40	80	50	90	60	90	70	80
10	70	40	60	30	50	20	40	10	30	-	40	10	50	40	60	30	70	40	50
11	50	80	40	70	30	60	20	50	10	40	-	30	10	40	20	50	30	50	40
14	90	60	80	50	70	40	60	30	50	20	40	10	30	-	40	10	50	20	30
15	70	100	60	90	50	80	40	70	30	60	20	50	10	40	-	30	10	30	20
17	80	100	70	90	60	80	50	70	40	60	30	50	20	40	10	30	-	20	10

Distance Calculations

➤ Walking distance = \sum number of passengers x distance

e.g. F1 assigned to gate 3,

Total walking distance =

$$5 \times 10 + 5 \times 40 + 0 \times 8 + \dots + 2 \times 90 + 1 \times 80 = 5,010 \text{ yards}$$

➤ Binary Decision Variable:

$$x_{i,j} = \begin{cases} 1 & \text{if flight } i \text{ is assigned to candidate gate } j \\ 0 & \text{otherwise} \end{cases}$$

Traveling Distance

Table 11.3 Traveling distances (yards)

Flight/gate	3	4	10	11	14	15	17
F1	5,010	4,390	3,820	4,870	5,060	6,650	7,090
F2	4,240	5,290	4,190	3,020	4,650	4,400	4,970
F3	5,610	5,950	4,930	4,270	4,910	4,950	5,320
F4	4,500	3,990	3,280	3,580	3,460	4,320	4,460
F5	2,950	2,720	3,060	3,490	3,620	4,330	4,530
F6	3,060	4,310	4,740	3,900	5,760	5,300	6,020
F7	4,680	4,380	3,290	3,620	3,970	4,960	5,220

► Minimize $5010x_{F1,3} + 4390x_{F1,4} + \dots + 5220x_{F7,17}$

► Constraints for Flight F1:

$$x_{F1,3} + x_{F1,4} + x_{F1,10} + x_{F1,11} + x_{F1,14} + x_{F1,15} + x_{F1,17} = 1$$

Similarly for the other 6 constraints for the other flights

► Additional Constraints for Gate 3:

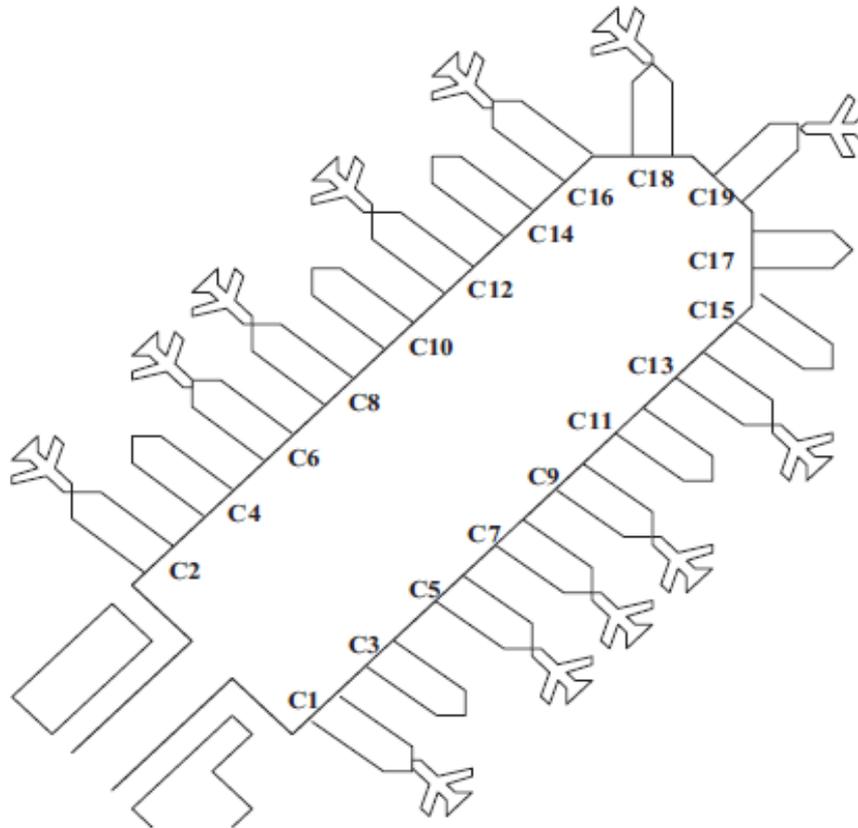
$$x_{F1,3} + x_{F2,3} + x_{F3,3} + x_{F4,3} + x_{F5,3} + x_{F6,3} + x_{F7,3} = 1$$

Similarly for the other constraints of the other six gates

- 49 binary decision variables and 14 constraints are generated for the integer linear programming.
- Optimal solution for the total walking distance among all passengers is 26,000 yards.

Case Study (Modify)

- Assume the gates 10 and 14 cannot be used for aircraft F1



Additional Constraints & Results

- Additional Constraints to the restriction of the assignment of gate 10 and 14 to flight F1.

$$X_{F1,10} = 0$$

$$X_{F1,14} = 0$$

- Running the model with new constraints

- Optimal solution for the total walking distance among all passengers is 26,700 yards.

Revised assignments of gates to flights

Flight	Gate assigned to
F1	4
F2	11
F3	15
F4	14
F5	17
F6	3
F7	10

- ▶ Transportation of baggage includes scheduling the number of baggage handlers, baggage trailers, delays, lost baggage and missed connections.

- ▶ Concept of baggage handling has been studied under different scopes:
 - Security purposes and detection of explosives (McLay et al. 2006 & Jasobson et al 2005)
 - Under gate assignment (Haghani and Chen 1998 and Lam, et al. 2002)
 - Adopting Radio Frequency Identification (RFID) (Maike 2008)

Mathematical Model for Baggage Handling



- Similar to gate assignment (Transit Passengers)
- Objective is to minimise the total traveling distance for transit bags
- Table below presents the amount of transit bags, mail, and cargo from each of the arriving aircraft to departing gates.

Table 11.6 Baggage flow from arriving flights to departing gates (units of baggage)

Flight	Departing gates																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
F1	19	28	11	8	30	25	33	5	49	14	38	38	14	23	17	4	20	44	8
F2	43	40	22	29	4	49	8	6	20	21	17	5	27	29	29	40	42	34	25
F3	22	17	36	45	22	28	17	23	18	44	12	8	41	48	25	11	27	47	28
F4	47	11	4	26	16	21	24	8	45	22	45	20	14	22	32	32	9	39	7
F5	3	24	46	38	48	7	24	33	29	43	7	21	45	47	28	11	17	3	23
F6	9	47	18	3	44	14	4	27	34	38	17	26	2	3	28	40	11	8	46
F7	46	34	48	42	26	12	40	49	18	36	24	6	18	9	2	10	14	47	9

Baggage Handling Case Study

- Assume baggage transported by baggage trailers from gates to gates on ramp and 5 bags per trailers.
- Convert the number of baggage flow to the number of trips that trailers need to make

e.g. 19 baggage units to be transported from F1 to gate 1:
 $[19/5] = 4$ (rounded-up integer)

- Consider the distance traveled by trailers to transport baggage :
Baggage transport distance = \sum number of trips x distance

Baggage Handling Case Study (Con't)



Table 11.7 Baggage flow in number of trips for trailers from arriving flights to departing gates

Flight	Gate																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
F1	4	6	3	2	6	5	7	1	10	3	8	8	3	5	4	1	4	9	2
F2	9	8	5	6	1	10	2	2	4	5	4	1	6	6	6	8	9	7	5
F3	5	4	8	9	5	6	4	5	4	9	3	2	9	10	5	3	6	10	6
F4	10	3	1	6	4	5	5	2	9	5	9	4	3	5	7	7	2	8	2
F5	1	5	10	8	10	2	5	7	6	9	2	5	9	10	6	3	4	1	5
F6	2	10	4	1	9	3	1	6	7	8	4	6	1	1	6	8	3	2	10
F7	10	7	10	9	6	3	8	10	4	8	5	2	4	2	1	2	3	10	2

Baggage Handling Case Study (Con't)



Table 11.8 Distance matrix for baggage trailers on the ramp (yards)

Gate	Departing Gates																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
3	15	60	-	45	15	60	30	75	45	90	60	105	75	120	90	135	105	135	120
4	68	17	51	-	68	17	85	34	102	51	119	68	136	85	153	102	153	119	136
10	112	64	96	48	80	32	64	16	48	-	64	16	80	64	96	48	112	64	80
11	65	104	52	91	39	78	26	65	13	52	-	39	13	52	26	65	39	65	52
14	135	90	120	75	105	60	90	45	75	30	60	15	45	-	60	15	75	30	45
15	98	140	84	126	70	112	56	98	42	84	28	70	14	56	-	42	14	42	28
17	112	140	98	126	84	112	70	98	56	84	42	70	28	56	14	42	-	28	14

Table 11.9 Baggage transport distances (yards)

Flight	Gate						
	3	4	10	11	14	15	17
F1	6,420	7,820	5,616	4,004	5,685	5,516	5,936
F2	7,965	8,636	6,992	5,707	6,525	6,762	6,986
F3	8,460	9,197	7,136	5,824	6,690	7,224	7,518
F4	7,005	8,296	6,064	4,537	6,015	5,922	6,426
F5	7,320	8,398	6,560	5,174	6,600	7,000	7,546
F6	6,975	7,480	5,328	4,719	5,565	5,978	6,244
F7	6,450	7,327	6,528	5,772	7,590	8,008	8,470

Case Study (Modify)

- Objective function: Minimize the total distance for both passenger and baggage traveling and transport distance.

$$\text{Total distance} = w_1(\text{Passenger traveling distances}) + w_2(\text{baggage transport distance})$$

- Assume higher weight to transport of baggage than to passenger traveling distances

$$w_1 = 1$$

$$w_2 = 3$$

- Minimize: $1(5010 \times F_{1,3} + 4290 \times F_{1,4} + \dots + 5220 \times F_{7,17}) + 3(6420 \times F_{1,3} + 7820 \times F_{1,4} + \dots + 8470 \times F_{7,17})$

Result

- Solution to gate assignment for both passenger and baggage transport

Flight	Gate assigned to
F1	11
F2	17
F3	14
F4	15
F5	3
F6	10
F7	4

Summary

- Mathematical model proposed by Bihr (1990)

Indices

i = index for arriving flights

j, k = index for gates

- Sets

F = set of arriving flights

G = set of available gates for arriving flights

K = set of departing gates

Summary (Con't)

Parameters

- $p_{i,k}$ = number of passengers arriving on flight i and departing from gate k
 dp_{kj} = distance units (in yards, meters, feet, etc) for passengers from gate k to gate j
 TP_{ij} = Total walking distance for all passengers on flight i assigned to arrival gate j
 $t_{i,k}$ = number of trips to transport baggage from flight i to departing gate k
 db_{kj} = distance units (in yards, meters, feet, etc.) to transport baggage on ramp from departing gate k to arriving gate j
 TB_{ij} = Total transport distance for all baggage on flight i assigned to arrival gate j
 w_1, w_2 = Weights assigned to total passenger walking and baggage transport distances respectively

TP_{ij} and TB_{ij} are calculated as follows:

$$TP_{i,j} = \sum_{k \in K} p_{i,k} \cdot dp_{k,j} \quad \text{for all } i \in F \text{ and } j \in G$$

$$TB_{i,j} = \sum_{k \in K} t_{i,k} \cdot db_{k,j} \quad \text{for all } i \in F \text{ and } j \in G$$

Summary (Con't)

Decision Variable

$$x_{i,j} = \begin{cases} 1 & \text{if flight } i \text{ is assigned to gate } j \\ 0 & \text{otherwise} \end{cases}$$

Objective Function

$$\text{Minimize } \sum_{i \in F} \sum_{j \in G} (w_1 TP_{i,j} x_{i,j} + w_2 TB_{i,j} x_{i,j})$$

Subject to

$$\sum_{j \in G} x_{i,j} = 1 \quad \text{for all } i \quad (11.1)$$

$$\sum_{i \in F} x_{i,j} = 1 \quad \text{for all } j \quad (11.2)$$

$$x_{i,j} \in \{0,1\} \quad \text{for all } i \text{ and } j$$

Key Reference

- Bazargan, M. (2010) Airline Operations and Scheduling. 2nd edition, Ashgate
 - Chapter 11 Gate Assignment

- Baggage Handling - STACK@EASE in HK

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- Baggage Handling - RFID

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